Hadroproduction data support tetraquark hypothesis for $\chi_{c1}(3872)$

Wai Kin Lai[1, 2, *] and Hee Sok Chung[3, †]

[1] State Key Laboratory of Nuclear Physics and Technology, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China

[2] Guangdong Basic Research Center of Excellence for Structure and Fundamental Interactions of Matter, Guangdong Provincial Key Laboratory of Nuclear Science, Guangzhou 510006, China

[3] Department of Mathematics and Physics, Gangneung-Wonju National University, Gangneung 25457, Korea

(Dated: August 26, 2025)

We show that the recently proposed tetraquark hypothesis for the nature of the $\chi_{c1}(3872)$ results in a formalism for inclusive production rates that has no unknown parameters. We employ this formalism to compute hadroproduction rates of $\chi_{c1}(3872)$ at the Large Hadron Collider, which agree with measured prompt and nonprompt cross sections. Thus, we find that the tetraquark hypothesis for $\chi_{c1}(3872)$ is well supported by hadroproduction data.

arXiv:2505.06910v2 [hep-ph] 25 Aug 2025

I. INTRODUCTION

The discovery of the narrow charmoniumlike state with a mass of about 3.872 GeV [1–4], now known as the $\chi_{c1}(3872)$, led to a renewed interest in exotic hadrons since their existence were prophesized in the 1960s [5, 6]. While this state carries quantum numbers that are consistent with a conventional charmonium [7, 8], it is widely considered a candidate for an exotic hadron [9]. There have been a number of hypotheses for its nature including it being a $D^{*0}\bar{D}^0$ molecule [10–15], a hybrid [16, 17], a tetraquark [18], and even a conventional charmonium [19, 20], but there seems to be no consensus on which the correct interpretation of this state is.

Inclusive production rates of $\chi_{c1}(3872)$ at hadron colliders has been considered a test ground for these hypotheses [21]. The prompt and nonprompt cross sections as functions of the transverse momentum p_T have been measured at the Large Hadron Collider (LHC) by the CMS [22], ATLAS [23], and LHCb [24] Collaborations. The prompt measurements are, for example, in conflict with the molecular [25] and the charmonium [26] hypotheses; while the mixing model in Refs. [27, 28] can explain the prompt rates, this comes at the cost of two nonperturbative unknowns that cannot be computed from first principles and must be determined from data.

Recently, in Refs. [29–31], the authors proposed a scenario based on the Born-Oppenheimer approximation of QCD where the $\chi_{c1}(3872)$ can arise as a $c\bar{c}q\bar{q}$ state, where the $c\bar{c}$ is predominantly in a color-octet state at short distances. While these hypotheses can be formulated in the Born-Oppenheimer effective field theory (BOEFT) approach [32–40], because some of the potentials used in Refs. [29–31] have not yet been obtained from first principles and instead are tuned to the spectrum, we may not yet consider this hypothesis a solid prediction of QCD. This calls for independent tests to be made to compare this scenario with measurements.

In this paper, we compute hadroproduction cross sections of the $\chi_{c1}(3872)$ under the assumption that it is a $c\bar{c}q\bar{q}$ tetraquark state as proposed in Refs. [29–31]. We find that, due to the color and angular momentum configuration of the $c\bar{c}$ at short distances in this hypothesis, the inclusive production rate takes a particularly simple form in the nonrelativistic QCD (NRQCD) factorization formalism [41], because the $\chi_{c1}(3872)$ is predominantly produced via a single color-octet state. Moreover, the nonperturbative NRQCD matrix element, which corresponds to the probability to find the $c\bar{c}$ inside the $\chi_{c1}(3872)$ in the color-octet state, can be computed from the $c\bar{c}$ wavefunction, which, in turn, can be determined from a Schrödinger equation. This leads to a formalism for inclusive production of $\chi_{c1}(3872)$ with no unknown parameters, which can be used as a stringent test of the tetraquark hypothesis for $\chi_{c1}(3872)$. We compare our calculation of prompt and nonprompt hadroproduction rates of $\chi_{c1}(3872)$ from pp collisions at the LHC and compare them with available data.

II. NRQCD FACTORIZATION FOR $\chi_{c1}(3872)$

In the NRQCD factorization formalism [41], the inclusive cross section of a $\chi_{c1}(3872)$ can be written as sums of products of the cross section of a $c\bar{c}$ in a specific color and angular momentum state and the nonperturbative NRQCD matrix element that corresponds to the probability to find the $c\bar{c}$ inside the $\chi_{c1}(3872)$ in the corresponding color and angular momentum state. The most important matrix element is the lowest-dimensional one that corresponds to the dominant $c\bar{c}$ Fock state of the $\chi_{c1}(3872)$. The contributions from matrix elements of higher dimensions, as well as matrix elements corresponding to subleading Fock states, are suppressed by powers of the scale Λ of the BOEFT divided by the charm quark mass m_c. In the case of the $\chi_{c1}(3872)$ in the tetraquark scenario, the dominant $c\bar{c}$ Fock state is the color-octet spin-triplet S-wave (${}^3S_1^{[8]}$) state [29–31], which corresponds to a matrix element of the lowest pos-

* samlai2357@gmail.com

† heesokchung@gwnu.ac.kr

sible dimension (dimension 3). Hence, we have

$$\sigma_{\chi_{c1}(3872)} = \sigma_{c\bar{c}(^3S_1^{[8]})} \langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]}) \rangle, \tag{1}$$

where $\sigma_{c\bar{c}(^3S_1^{[8]})}$ is the perturbatively calculable production rate of a $c\bar{c}$ in the $^3S_1^{[8]}$ state and $\langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]}) \rangle$ is the corresponding NRQCD matrix element. A striking feature of this formula is that only one channel contributes to the cross section at leading power in the nonrelativistic expansion. This is because the matrix elements that correspond to Fock states other than the $^3S_1^{[8]}$ one are always of higher dimensions; for example, the color-octet D-wave component contributes through a dimension-7 matrix element, which is suppressed by at least $(\Lambda/m_c)^4$ compared to the $^3S_1^{[8]}$ contribution. The color-singlet P-wave component contributes through dimension-5 matrix elements, which are suppressed by at least $(\Lambda/m_c)^2$ compared to the leading contribution; this is further suppressed due to the fact that the regular charmonium component is small in the $\chi_{c1}(3872)$ state [31]. This is in stark contrast with the case of J/ψ or $\psi(2S)$ production phenomenology, where, although only color-singlet matrix elements appear at leading power in the nonrelativistic expansion, it is necessary to include contributions from color-octet matrix elements with suppressed power counts in the factorization formula, because production rates of a color-octet $c\bar{c}$ are much greater than that of a color singlet and can dominate the cross section. This does not happen in the case of $\chi_{c1}(3872)$, because the $^3S_1^{[8]}$ channel already has a large $c\bar{c}$ production rate and there is no source of dynamical enhancement from power-suppressed channels.

III. PREDICTION OF NONPROMPT FRACTION

We can already make predictions based on Eq. (1) by using the fact that, since the formula involves only one channel, the NRQCD matrix element cancels in cross section ratios. The nonprompt fraction, which is the ratio of the nonprompt cross section to the sum of the prompt and nonprompt cross sections, has been measured by CMS [22] and ATLAS [23]. We can compute this fraction by using Eq. (1) as

$$\frac{\sigma^{\rm nonprompt}_{\chi_{c1}(3872)}}{\sigma^{\rm prompt}_{\chi_{c1}(3872)} + \sigma^{\rm nonprompt}_{\chi_{c1}(3872)}} = \frac{\sigma^{\rm nonprompt}_{c\bar{c}(^3S_1^{[8]})}}{\sigma^{\rm prompt}_{c\bar{c}(^3S_1^{[8]})} + \sigma^{\rm nonprompt}_{c\bar{c}(^3S_1^{[8]})}}. \tag{2}$$

Here, $\sigma^{\rm nonprompt}_{c\bar{c}(^3S_1^{[8]})}$ is given by

$$\sigma^{\rm nonprompt}_{c\bar{c}(^3S_1^{[8]})} = 2 \times \sigma_b \times C(b \to c\bar{c}(^3S_1^{[8]}) + X), \tag{3}$$

where the factor 2 counts for the fact that the nonprompt cross section comes from both b and $\bar{b}$ decays, σ_b is the production rate of a b quark, and $C(b \to c\bar{c}(^3S_1^{[8]}) + X)$ is the NRQCD short-distance coefficient for the branching fraction $\text{Br}(b \to \chi_{c1}(3872) + X) = C(b \to c\bar{c}(^3S_1^{[8]}) + X) \times \langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]}) \rangle$. This short-distance coefficient has been computed at next-to-leading order (NLO) in Ref. [42], which gives $C(b \to c\bar{c}(^3S_1^{[8]}) + X) = 0.223$ GeV^{-3} in the 't Hooft–Veltman scheme. We take the fixed order+next-to-leading log (FONLL) result for σ_b [43, 44], in which we include the correction from the nonperturbative fragmentation function. We compute the prompt cross section in two ways. In fixed-order (FO) perturbation theory, we compute the prompt $c\bar{c}(^3S_1^{[8]})$ cross section at NLO by using the FDCHQHP package [45]. Because the prompt cross section is dominated by the fragmentation process which involves singularities at the kinematical threshold, threshold logarithms that appear in radiative corrections can become significant; therefore, we also compute the *resummed* prompt cross section by using the results of Ref. [46], where the threshold double logarithms, as well as the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) logarithms [47–50], are resummed to all orders in perturbation theory. We note that the DGLAP logarithms are also resummed in the FONLL calculation of σ_b. The numerical effect of threshold resummation in the prompt cross section is similar to the inclusion of the nonperturbative fragmentation function in FONLL, as both have the effect of replacing the singular distribution in the fragmentation function with a smooth one. Neither resummation of threshold or DGLAP logarithms are included in the FO calculation. We choose $m_c = 1.5$ GeV, which is the default choice in Refs. [43–45], and set the scales to p_T.

We compare our results for the nonprompt fraction with CMS and ATLAS data in Fig. 1. We include uncertainties from varying the scales by factors 1/2 and 2, the uncertainty from the b quark mass in the FONLL calculation, and estimate the uncertainty in the theoretical calculation of the branching fraction from the dependence on the γ_5 scheme by the difference between the results in the 't Hooft–Veltman and the naïve dimensional regularization schemes, which amounts to about 22%. The uncertainties are added in quadrature. We

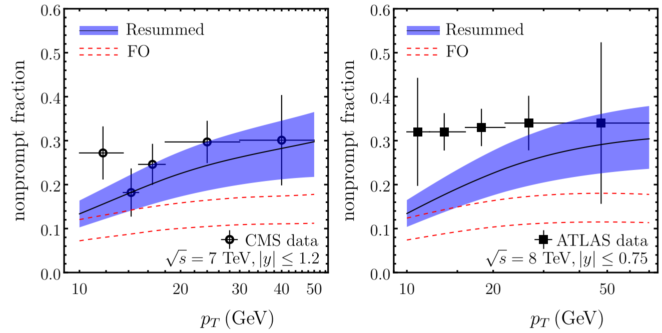

FIG. 1. Nonprompt fractions for $\chi_{c1}(3872)$ production computed from Eq. (2) compared to CMS (left) and ATLAS (right) data.

see that, when we include resummation when computing both the prompt and nonprompt cross sections, our predictions are in fair agreement with the CMS data and also with ATLAS data at large p_T. The FO results, where effects of resummation are neglected in the prompt contribution, are systematically below data. The fact that our prediction for the nonprompt fraction that requires no unknown nonperturbative parameters agree with measurements makes a very compelling case for the tetraquark hypothesis for $\chi_{c1}(3872)$.

IV. PROMPT AND NONPROMPT CROSS SECTIONS

A. NRQCD matrix element for $\chi_{c1}(3872)$

Another important test of the scenario in Refs. [29–31] is to see if we can reproduce the absolute cross sections at hadron colliders, which require knowledge of the ${}^3S_1^{[8]}$ matrix element. By using the techniques developed in Refs. [51–54], we compute this matrix element in terms of the $c\bar{c}$ wavefunction of the $\chi_{c1}(3872)$. The calculation is almost identical to the case of the ${}^3S_1^{[1]}$ matrix element for J/ψ, except that for $\chi_{c1}(3872)$, the $c\bar{c}$ is in a color-octet state. The ${}^3S_1^{[8]}$ matrix element is defined by the vacuum expectation value of the operator [41, 55]

$$\chi^\dagger \sigma^i T^a \psi \Phi_\ell^{\dagger ab} \mathcal{P}_{\chi_{c1}(3872)} \Phi_\ell^{bc} \psi^\dagger \sigma^i T^c \chi, \tag{4}$$

where ψ and χ are Pauli spinor fields that annihilate and create a charm quark and antiquark, respectively, σ^i are Pauli matrices, T^a are color matrices, and $\Phi_\ell = \mathcal{P}\exp[-ig\int_0^\infty d\lambda \ell\cdot A^{\rm adj}(\lambda\ell)]$ is a path-ordered Wilson line in the adjoint representation defined along a lightlike direction ℓ. The $\mathcal{P}_{\chi_{c1}(3872)}$ is an operator that projects onto states that include a $\chi_{c1}(3872)$ at rest. This operator can be written as

$$\mathcal{P}_{\chi_{c1}(3872)} = \int d^{d-1}x_1 d^{d-1}x_2 d^{d-1}x_1' d^{d-1}x_2' \sum_{n\in\mathbb{O}} \phi(\boldsymbol{x}_1' - \boldsymbol{x}_2') |\underline{\mathrm{n}}; \boldsymbol{x}_1', \boldsymbol{x}_2'\rangle \langle \underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2| \phi^*(\boldsymbol{x}_1 - \boldsymbol{x}_2), \tag{5}$$

where d is the number of spacetime dimensions, $\phi(\boldsymbol{x}_1 - \boldsymbol{x}_2)$ is the unit-normalized $c\bar{c}$ wavefunction with $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ the position of the c and $\bar{c}$, respectively, and $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ is the NRQCD state of an excitation we denote with n. Here, we used the heavy-quark spin symmetry to lift the correlation between the angular momentum of the $c\bar{c}$ and the light degrees of freedom. The sum over n is restricted to the subspace $\mathbb{O}$ where the $c\bar{c}$ is in the color-octet state in the limit $\boldsymbol{x}_1 - \boldsymbol{x}_2 \to 0$. To compute the vacuum expectation value of Eq. (4), we need the matrix element of $\Phi_\ell^{bc}\psi^\dagger\sigma^i T^c \chi$ between the QCD vacuum and the state $\langle \underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2|$. By using the techniques used in Refs. [51–54], we obtain at leading power in the nonrelativistic expansion

$$\langle \underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2 | \Phi_\ell^{bc} \psi^\dagger \boldsymbol{\sigma} T^c \chi |0\rangle = \delta^{(d-1)}(\boldsymbol{x} - \boldsymbol{x}_1) \times \delta^{(d-1)}(\boldsymbol{x} - \boldsymbol{x}_2) \langle n; \boldsymbol{x}_1, \boldsymbol{x}_2 | \Phi_\ell^{bc} \boldsymbol{\sigma} T^d \Phi_0^{dc} |0\rangle, \tag{6}$$

where $|0\rangle$ is the QCD vacuum, $\boldsymbol{x}$ is the spatial position of the operator on the left side, $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle = \psi^\dagger(\boldsymbol{x}_1)\chi(\boldsymbol{x}_2)|n; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$, the spin and fundamental color indices of $\boldsymbol{\sigma}$ and T^d are fully contracted with the ones implicit in the state $\langle n; \boldsymbol{x}_1, \boldsymbol{x}_2|$, and $\Phi_0 = \mathcal{P}\exp[-ig\int_0^\infty d\lambda A_0^{\rm adj}(\lambda, \boldsymbol{0})]$ is the temporal Wilson line in the adjoint representation. Because of this, this matrix element vanishes unless the $c\bar{c}$ is in the color-octet state at $\boldsymbol{x}_1 - \boldsymbol{x}_2 \to \boldsymbol{0}$. Therefore, we can lift the restriction $n \in \mathbb{O}$ in the sum and use the completeness relation $\sum_n |n; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle\langle n; \boldsymbol{x}_1, \boldsymbol{x}_2| = 1$, which leads to

$$\langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]})\rangle = 2T_F(d-1)\mathcal{S}|\phi(\boldsymbol{0})|^2, \tag{7}$$

where the factors $2(d-1)$ and $T_F = 1/2$ come from the trace over spin and color, respectively, $\phi(\boldsymbol{0})$ is the $\chi_{c1}(3872)$ wavefunction at the origin, and $\mathcal{S}$ is a dimensionless quantity defined by the vacuum expectation value of adjoint Wilson lines:

$$\mathcal{S} = \langle 0|[\Phi_0^{ca}\Phi_\ell^{ba}]^\dagger \Phi_0^{cd}\Phi_\ell^{bd}|0\rangle. \tag{8}$$

In perturbation theory, $\mathcal{S} = 1$ at tree level, and the NLO correction vanishes [46]. As we work at NLO accuracy, we take the perturbative QCD result $\mathcal{S} = 1 + O(\alpha_s^2)$. We then obtain

$$\langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]})\rangle = 3|\phi(\boldsymbol{0})|^2. \tag{9}$$

Because we can compute the wavefunction ϕ by solving a Schrödinger equation, a potential model for the tetraquark completely determines the value of the ${}^3S_1^{[8]}$ matrix element at leading power in the nonrelativistic expansion. Therefore, the proposed scenario in Refs. [29–31] leads to a formalism for inclusive production rates of $\chi_{c1}(3872)$ that has no unknown parameters. An analogous formalism would hold, for example, for production of hybrid states with a $c\bar{c}$ component in the S-wave state.

B. Prediction of cross sections

In order to compute absolute cross sections we must determine the ${}^3S_1^{[8]}$ matrix element from $|\phi(\boldsymbol{0})|^2$, which can be obtained by solving the Schrödinger equation. In practice, however, we find that the result varies depending on the choice of potentials and parameters of the Schrödinger equation; we obtain values of $|\phi(\boldsymbol{0})|^2$ that range from 0.69×10^{-3} to 2.0×10^{-3} GeV^3 from Refs. [29–31]. As our knowledge of the tetraquark potential is currently limited, for obtaining phenomenological results in this work we choose to fix $|\phi(\boldsymbol{0})|^2$ against the experimental value $\mathrm{Br}(b \to \chi_{c1}(3872) + X) \times \mathrm{Br}(\chi_{c1}(3872) \to$

$J/\psi\pi^+\pi^-) = (4.3 \pm 0.5) \times 10^{-5}$ from LHCb [24] and the Particle Data Group value $\mathrm{Br}(\chi_{c1}(3872) \to J/\psi\pi^+\pi^-) = (3.5\pm0.9)\times10^{-2}$ [56]. This corresponds to the central values $|\phi(\mathbf{0})|^2 = 1.8 \times 10^{-3}$ GeV^3 and $\langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\rangle = 5.5 \times 10^{-3}$ GeV^{-3}, which lie within the range of potential model results. We consider the uncertainties in the $c\bar{c}$ cross sections by varying the scales by factors 1/2 and 2. The uncertainty in $|\phi(\mathbf{0})|^2$ coming from the input parameters and the theoretical uncertainty in $C(b \to c\bar{c}(^3S_1^{[8]}) + X)$ amount to about 25%. Because our result for $\langle \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\rangle$ can receive corrections of higher orders in the nonrelativistic expansion, we estimate the total uncertainty in the matrix element to be 40% of the central value.

We first compute the prompt cross sections from pp collisions at the LHC. The calculation of the $c\bar{c}$ cross sections are done in the same way as in the nonprompt fractions. The comparison with CMS, ATLAS, and LHCb data are shown in Fig. 2. The theoretical uncertainties come from scale variations and the $^3S_1^{[8]}$ matrix element. Our resummed results are in agreement with data, with the exception of ATLAS data for the two largest p_T bins. The FO results tend to overestimate data, although the FO and resummed results are consistent within uncertainties. The difference between FO and resummed results diminish with decreasing p_T. In the LHCb case, we do not show resummed results below $p_T < 8$ GeV, because the cross section no longer becomes dominated by fragmentation.

We then compute the nonprompt cross sections from pp collisions at the LHC. We again use the FONLL results for the b production rates [43–45]. Our results compared with ATLAS and LHCb data are shown in Fig. 3. We see that our results are in good agreement with nonprompt measurements.

C. Comparison with other model predictions

As we have stated earlier, the molecular [25] and conventional charmonium [26] hypotheses lead to predictions of the prompt cross section that disagree with measurements [22]. In the mixing model [27, 28], where the production rate of the $\chi_{c1}(3872)$ is dominated by the conventional charmonium component, cross section predictions require determination of two nonperturbative unknowns: the $^3S_1^{[8]}$ matrix element of the P-wave charmonium and $Z_{c\bar{c}}$, the amount of the conventional charmonium admixture in the $\chi_{c1}(3872)$ state. In Ref. [28], these unknowns have been obtained from fits to prompt cross section measurements. We find that they lead to values of $\mathrm{Br}(b \to \chi_{c1}(3872)+X)\mathrm{Br}(\chi_{c1}(3872) \to J/\psi\pi^+\pi^-)$ less than about 1.1×10^{-5}, which is more than 3 times smaller than the LHCb result [24]. Hence, mixing-model predictions for nonprompt production rates and nonprompt fractions are generally below data.

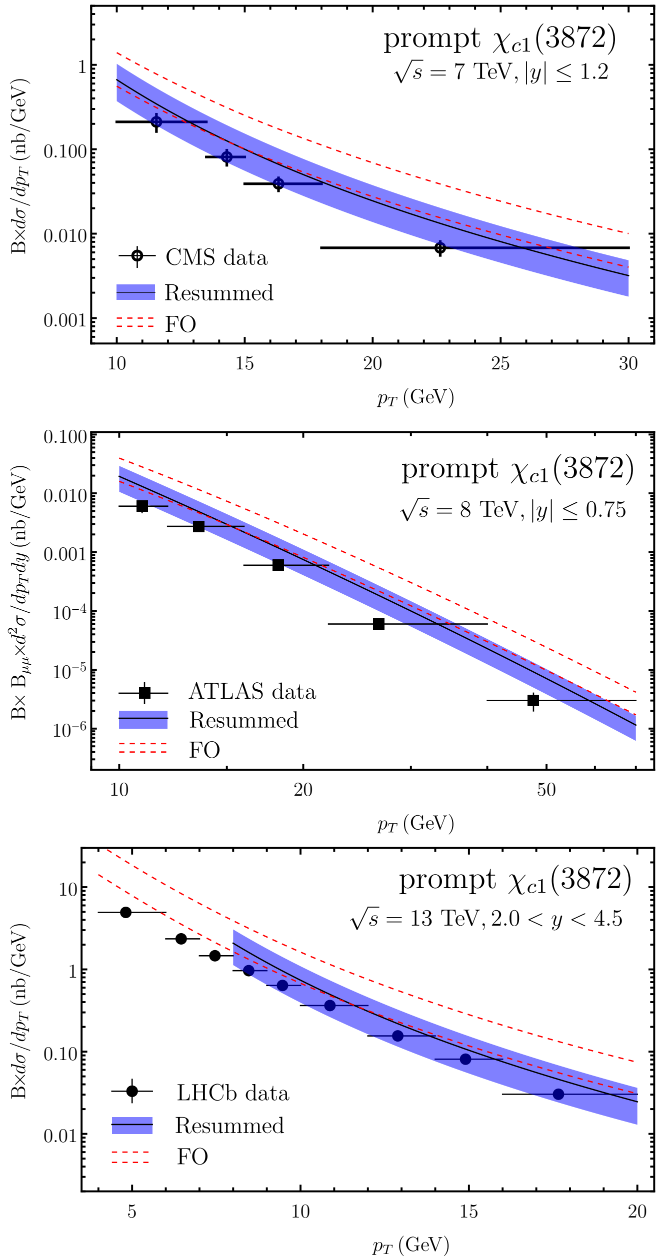

FIG. 2. Prompt production rates of $\chi_{c1}(3872)$ computed in the tetraquark hypothesis compared to CMS, ATLAS, and LHCb data. $\mathrm{B} \equiv \mathrm{Br}(\chi_{c1}(3872) \to J/\psi\pi^+\pi^-)$ and $\mathrm{B}_{\mu\mu} \equiv \mathrm{Br}(J/\psi \to \mu^+\mu^-)$.

V. CONCLUSIONS

In this work, we showed that the tetraquark hypothesis for the nature of $\chi_{c1}(3872)$ proposed in Refs. [29–31] leads to a formalism for its inclusive production rates that has no unknown parameters. Unlike the case of conventional quarkonium, the production rate of $\chi_{c1}(3872)$ in this tetraquark hypothesis involves only one color-octet channel in the nonrelativistic QCD factorization formalism, and the nonperturbative matrix element can be deter-

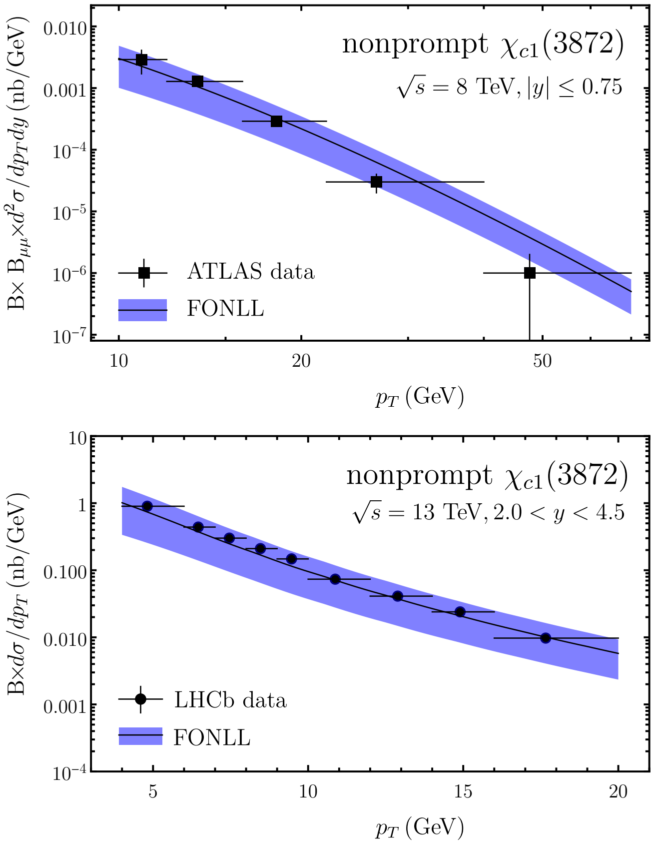

FIG. 3. Nonprompt production rates of $\chi_{c1}(3872)$ computed in the tetraquark hypothesis compared to ATLAS and LHCb data. $\mathrm{B} \equiv \mathrm{Br}(\chi_{c1}(3872) \to J/\psi\pi^+\pi^-)$ and $\mathrm{B}_{\mu\mu} \equiv \mathrm{Br}(J/\psi \to \mu^+\mu^-)$.

mined from the $c\bar{c}$ wavefunction. We computed prompt and nonprompt production rates of $\chi_{c1}(3872)$ from pp collisions at the LHC, which are generally in agreement with CMS, ATLAS, and LHCb measurements. This is a strong implication that hadroproduction data support the tetraquark hypothesis for $\chi_{c1}(3872)$.

Currently, the precision of the theory prediction for absolute cross sections is limited by the lack of knowledge in the potentials of the BOEFT. While it seems promising that the nonperturbative matrix element fixed against LHCb data [24] used in this work is within the range of potential model calculations based on Refs. [29–31], more theoretical progress will need to be made in order to reduce model dependence. We also note that our theoretical results tend to overshoot data at the largest p_T bins, which may signal the need for resummation of threshold logarithms beyond leading double logarithmic level [46].

If it turns out that the $\chi_{c1}(3872)$ is indeed a tetraquark as proposed in Refs. [29–31], the production mechanism we established in this work would open new possibilities to probe QCD interactions through production of $\chi_{c1}(3872)$ in colliders. Although the idea that a $c\bar{c}$ meson could be produced predominantly through the ${}^3S_1^{[8]}$ channel is not new [57], it is now well known that the old idea of ${}^3S_1^{[8]}$ dominance for J/ψ and $\psi(2S)$ production fails to describe large-p_T hadroproduction data, especially for polarized cross sections, and it is likely that the mechanism for production of conventional charmonium is much more involved [58, 59]. In contrast, $\chi_{c1}(3872)$ production processes could directly probe perturbative QCD through production of $c\bar{c}$ in the ${}^3S_1^{[8]}$ state. For example, we can already see that the inclusion of important radiative corrections including threshold and DGLAP logarithms to all orders in perturbation theory pioneered in Refs. [46, 60–62] significantly improves the QCD description of the nonprompt fraction, compared to strictly fixed-order calculations. We anticipate that our knowledge of perturbative $c\bar{c}$ production rates honed through decades of quarkonium production phenomenology can finally be turned into a scalpel to dissect QCD through hadroproduction of $\chi_{c1}(3872)$.

ACKNOWLEDGMENTS

We thank Abhishek Mohapatra for useful discussions. The work of H. S. C. is supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (Grant No. RS-2023-00248313). W. K. L. is supported by the National Natural Science Foundation of China (NSFC) under Grant No. 12035007 and by the Guangdong Major Project of Basic and Applied Basic Research No. 2020B0301030008.

[1] S. K. Choi et al. (Belle), Observation of a narrow charmonium-like state in exclusive $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ decays, Phys. Rev. Lett. **91**, 262001 (2003), arXiv:hep-ex/0309032.

[2] D. Acosta et al. (CDF), Observation of the narrow state $X(3872) \to J/\psi\pi^+\pi^-$ in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV, Phys. Rev. Lett. **93**, 072001 (2004), arXiv:hep-ex/0312021.

[3] V. M. Abazov et al. (D0), Observation and properties of the $X(3872)$ decaying to $J/\psi\pi^+\pi^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, Phys. Rev. Lett. **93**, 162002 (2004), arXiv:hep-ex/0405004.

[4] B. Aubert et al. (BaBar), Study of the $B \to J/\psi K^-\pi^+\pi^-$ decay and measurement of the $B \to X(3872)K^-$ branching fraction, Phys. Rev. D **71**, 071103 (2005), arXiv:hep-ex/0406022.

[5] M. Gell-Mann, A Schematic Model of Baryons and Mesons, Phys. Lett. **8**, 214 (1964).

[6] G. Zweig, An SU(3) model for strong interaction symmetry and its breaking. Version 1 10.17181/CERN-TH-401 (1964).

[7] R. Aaij et al. (LHCb), Determination of the X(3872) meson quantum numbers, Phys. Rev. Lett. **110**, 222001 (2013), arXiv:1302.6269 [hep-ex].

[8] R. Aaij et al. (LHCb), Quantum numbers of the $X(3872)$ state and orbital angular momentum in its $\rho^0 J\psi$ decay, Phys. Rev. D **92**, 011102 (2015), arXiv:1504.06339 [hep-ex].

[9] T. Barnes and S. Godfrey, Charmonium options for the X(3872), Phys. Rev. D **69**, 054008 (2004), arXiv:hep-ph/0311162.

[10] N. A. Tornqvist, Isospin breaking of the narrow charmonium state of Belle at 3872-MeV as a deuson, Phys. Lett. B **590**, 209 (2004), arXiv:hep-ph/0402237.

[11] M. B. Voloshin, Interference and binding effects in decays of possible molecular component of X(3872), Phys. Lett. B **579**, 316 (2004), arXiv:hep-ph/0309307.

[12] F. E. Close and P. R. Page, The D*0 anti-D0 threshold resonance, Phys. Lett. B **578**, 119 (2004), arXiv:hep-ph/0309253.

[13] C.-Y. Wong, Molecular states of heavy quark mesons, Phys. Rev. C **69**, 055202 (2004), arXiv:hep-ph/0311088.

[14] E. Braaten and M. Kusunoki, Low-energy universality and the new charmonium resonance at 3870-MeV, Phys. Rev. D **69**, 074005 (2004), arXiv:hep-ph/0311147.

[15] E. S. Swanson, Short range structure in the X(3872), Phys. Lett. B **588**, 189 (2004), arXiv:hep-ph/0311229.

[16] F. E. Close and S. Godfrey, Charmonium hybrid production in exclusive B meson decays, Phys. Lett. B **574**, 210 (2003), arXiv:hep-ph/0305285.

[17] B. A. Li, Is X(3872) a possible candidate of hybrid meson, Phys. Lett. B **605**, 306 (2005), arXiv:hep-ph/0410264.

[18] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Diquark-antidiquarks with hidden or open charm and the nature of X(3872), Phys. Rev. D **71**, 014028 (2005), arXiv:hep-ph/0412098.

[19] E. J. Eichten, K. Lane, and C. Quigg, B Meson Gateways to Missing Charmonium Levels, Phys. Rev. Lett. **89**, 162002 (2002), arXiv:hep-ph/0206018.

[20] E. J. Eichten, K. Lane, and C. Quigg, Charmonium levels near threshold and the narrow state $X(3872) \to \pi^+\pi^- J/\psi$, Phys. Rev. D **69**, 094019 (2004), arXiv:hep-ph/0401210.

[21] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa, and C. Sabelli, Is the X(3872) Production Cross Section at Tevatron Compatible with a Hadron Molecule Interpretation?, Phys. Rev. Lett. **103**, 162001 (2009), arXiv:0906.0882 [hep-ph].

[22] S. Chatrchyan et al. (CMS), Measurement of the $X(3872)$ Production Cross Section Via Decays to $J/\psi\pi^+\pi^-$ in pp collisions at $\sqrt{s} = 7$ TeV, JHEP **04**, 154, arXiv:1302.3968 [hep-ex].

[23] M. Aaboud et al. (ATLAS), Measurements of $\psi(2S)$ and $X(3872) \to J/\psi\pi^+\pi^-$ production in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, JHEP **01**, 117, arXiv:1610.09303 [hep-ex].

[24] R. Aaij et al. (LHCb), Measurement of $\chi_{c1}(3872)$ production in proton-proton collisions at $\sqrt{s} = 8$ and 13 TeV, JHEP **01**, 131, arXiv:2109.07360 [hep-ex].

[25] P. Artoisenet and E. Braaten, Production of the X(3872) at the Tevatron and the LHC, Phys. Rev. D **81**, 114018 (2010), arXiv:0911.2016 [hep-ph].

[26] M. Butenschoen, Z.-G. He, and B. A. Kniehl, NLO NRQCD disfavors the interpretation of X(3872) as $\chi_{c1}(2P)$, Phys. Rev. D **88**, 011501 (2013), arXiv:1303.6524 [hep-ph].

[27] C. Meng, Y.-J. Gao, and K.-T. Chao, B $\to \chi_{c1}$(1P,2P)K decays in QCD factorization and X(3872), Phys. Rev. D **87**, 074035 (2013), arXiv:hep-ph/0506222.

[28] C. Meng, H. Han, and K.-T. Chao, X(3872) and its production at hadron colliders, Phys. Rev. D **96**, 074014 (2017), arXiv:1304.6710 [hep-ph].

[29] B. Grinstein, L. Maiani, and A. D. Polosa, Radiative decays of X(3872) discriminate between the molecular and compact interpretations, Phys. Rev. D **109**, 074009 (2024), arXiv:2401.11623 [hep-ph].

[30] E. Braaten and R. Bruschini, Exotic hidden-heavy hadrons and where to find them, Phys. Lett. B **863**, 139386 (2025), arXiv:2409.08002 [hep-ph].

[31] N. Brambilla, A. Mohapatra, T. Scirpa, and A. Vairo, The nature of $\chi_{c1}(3872)$ and $T_{cc}^+(3875)$, (2024), arXiv:2411.14306 [hep-ph].

[32] M. Born and R. Oppenheimer, Zur Quantentheorie der Molekeln, Annalen Phys. **389**, 457 (1927).

[33] K. J. Juge, J. Kuti, and C. J. Morningstar, Ab initio study of hybrid anti-b g b mesons, Phys. Rev. Lett. **82**, 4400 (1999), arXiv:hep-ph/9902336.

[34] E. Braaten, C. Langmack, and D. H. Smith, Born-Oppenheimer Approximation for the XYZ Mesons, Phys. Rev. D **90**, 014044 (2014), arXiv:1402.0438 [hep-ph].

[35] M. Berwein, N. Brambilla, J. Tarrús Castellà, and A. Vairo, Quarkonium Hybrids with Nonrelativistic Effective Field Theories, Phys. Rev. D **92**, 114019 (2015), arXiv:1510.04299 [hep-ph].

[36] R. Oncala and J. Soto, Heavy Quarkonium Hybrids: Spectrum, Decay and Mixing, Phys. Rev. D **96**, 014004 (2017), arXiv:1702.03900 [hep-ph].

[37] N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà, and A. Vairo, Spin structure of heavy-quark hybrids, Phys. Rev. D **99**, 014017 (2019), [Erratum: Phys.Rev.D 101, 099902 (2020)], arXiv:1805.07713 [hep-ph].

[38] N. Brambilla, W. K. Lai, J. Segovia, and J. Tarrús Castellà, QCD spin effects in the heavy hybrid potentials and spectra, Phys. Rev. D **101**, 054040 (2020), arXiv:1908.11699 [hep-ph].

[39] L. Maiani, A. D. Polosa, and V. Riquer, Hydrogen bond of QCD, Phys. Rev. D **100**, 014002 (2019), arXiv:1903.10253 [hep-ph].

[40] M. Berwein, N. Brambilla, A. Mohapatra, and A. Vairo, Hybrids, tetraquarks, pentaquarks, doubly heavy baryons, and quarkonia in Born-Oppenheimer effective theory, Phys. Rev. D **110**, 094040 (2024), arXiv:2408.04719 [hep-ph].

[41] G. T. Bodwin, E. Braaten, and G. P. Lepage, Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium, Phys. Rev. D **51**, 1125 (1995), [Erratum: Phys.Rev.D 55, 5853 (1997)], arXiv:hep-ph/9407339.

[42] M. Beneke, F. Maltoni, and I. Z. Rothstein, QCD analysis of inclusive B decay into charmonium, Phys. Rev. D **59**, 054003 (1999), arXiv:hep-ph/9808360.

[43] M. Cacciari, M. Greco, and P. Nason, The p_T spectrum in heavy-flavour hadroproduction., JHEP **05**, 007, arXiv:hep-ph/9803400.

[44] M. Cacciari, S. Frixione, and P. Nason, The p(T) spectrum in heavy flavor photoproduction, JHEP **03**, 006,

arXiv:hep-ph/0102134.
[45] L.-P. Wan and J.-X. Wang, FDCHQHP: A Fortran package for heavy quarkonium hadroproduction, Comput. Phys. Commun. **185**, 2939 (2014), arXiv:1405.2143 [hep-ph].
[46] H. S. Chung, U.-R. Kim, and J. Lee, Resummation of Threshold Double Logarithms in Hadroproduction of Heavy Quarkonium, Phys. Rev. Lett. **134**, 071902 (2025), arXiv:2408.04255 [hep-ph].
[47] V. N. Gribov and L. N. Lipatov, Deep inelastic e p scattering in perturbation theory, Sov. J. Nucl. Phys. **15**, 438 (1972).
[48] L. N. Lipatov, The parton model and perturbation theory, Yad. Fiz. **20**, 181 (1974), Sov. J. Nucl. Phys. **20**, 94 (1975).
[49] Y. L. Dokshitzer, Calculation of the Structure Functions for Deep Inelastic Scattering and e+ e- Annihilation by Perturbation Theory in Quantum Chromodynamics., Sov. Phys. JETP **46**, 641 (1977).
[50] G. Altarelli and G. Parisi, Asymptotic Freedom in Parton Language, Nucl. Phys. B **126**, 298 (1977).
[51] N. Brambilla, H. S. Chung, and A. Vairo, Inclusive Hadroproduction of P-Wave Heavy Quarkonia in Potential Nonrelativistic QCD, Phys. Rev. Lett. **126**, 082003 (2021), arXiv:2007.07613 [hep-ph].
[52] N. Brambilla, H. S. Chung, and A. Vairo, Inclusive production of heavy quarkonia in pNRQCD, JHEP **09**, 032, arXiv:2106.09417 [hep-ph].
[53] N. Brambilla, H. S. Chung, A. Vairo, and X.-P. Wang, Production and polarization of S-wave quarkonia in potential nonrelativistic QCD, Phys. Rev. D **105**, L111503 (2022), arXiv:2203.07778 [hep-ph].
[54] N. Brambilla, H. S. Chung, A. Vairo, and X.-P. Wang, Inclusive production of J/ψ, ψ(2S), and Υ states in pNRQCD, JHEP **03**, 242, arXiv:2210.17345 [hep-ph].
[55] G. C. Nayak, J.-W. Qiu, and G. F. Sterman, Fragmentation, factorization and infrared poles in heavy quarkonium production, Phys. Lett. B **613**, 45 (2005), arXiv:hep-ph/0501235.
[56] S. Navas et al. (Particle Data Group), Review of Particle Physics, Phys. Rev. D **110**, 030001 (2024).
[57] E. Braaten and S. Fleming, Color octet fragmentation and the psi-prime surplus at the Tevatron, Phys. Rev. Lett. **74**, 3327 (1995), arXiv:hep-ph/9411365.
[58] H. S. Chung, Review of quarkonium production: status and prospects, PoS **Confinement2018**, 007 (2018), arXiv:1811.12098 [hep-ph].
[59] H. S. Chung, Quarkonium production and polarization: where do we stand?, EPJ Web Conf. **274**, 01011 (2022), arXiv:2211.10201 [hep-ph].
[60] G. T. Bodwin, H. S. Chung, U.-R. Kim, and J. Lee, Fragmentation contributions to J/ψ production at the Tevatron and the LHC, Phys. Rev. Lett. **113**, 022001 (2014), arXiv:1403.3612 [hep-ph].
[61] G. T. Bodwin, K.-T. Chao, H. S. Chung, U.-R. Kim, J. Lee, and Y.-Q. Ma, Fragmentation contributions to hadroproduction of prompt J/ψ, χ_{cJ}, and $\psi(2S)$ states, Phys. Rev. D **93**, 034041 (2016), arXiv:1509.07904 [hep-ph].
[62] A.-P. Chen, X.-B. Jin, Y.-Q. Ma, and C. Meng, Fragmentation function of g $\to Q\overline{Q}(^3S_1^{[8]})$ in soft gluon factorization and threshold resummation, JHEP **06**, 046, arXiv:2103.15121 [hep-ph].