AUGMENTED REGRESSION MODELS USING NEUROCHAOS LEARNING

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ABSTRACT

This study presents novel Augmented Regression Models using Neurochaos Learning (NL), where *Tracemean* features derived from the Neurochaos Learning framework are integrated with traditional regression algorithms —- *Linear Regression, Ridge Regression, Lasso Regression*, and *Support Vector Regression (SVR)*. Our approach was evaluated using ten diverse real-life datasets and a synthetically generated dataset of the form $y = mx + c + \epsilon$. Results show that incorporating the *Tracemean* feature (mean of the chaotic neural traces of the neurons in the NL architecture) significantly enhances regression performance, particularly in Augmented Lasso Regression and Augmented SVR, where six out of ten real-life datasets exhibited improved predictive accuracy. Among the models, Augmented Chaotic Ridge Regression achieved the highest average performance boost (11.35%). Additionally, experiments on the simulated dataset demonstrated that the Mean Squared Error (MSE) of the augmented models consistently decreased and converged towards the Minimum Mean Squared Error (MMSE) as the sample size increased. This work demonstrates the potential of chaos-inspired features in regression tasks, offering a pathway to more accurate and computationally efficient prediction models.

***K*eywords** Linear Regression · Neurochaos · Tracemean · Ridge Regression · Lassso · Support Vector Regression

1 Introduction

In today's world, we are surrounded by an abundance of data generated from various domains, including healthcare, finance, agriculture, and more. Extracting meaningful insights from this data has the potential to bring transformative changes. Among the most critical tasks in data science is prediction—identifying the underlying function or model that governs data generation. Accurate prediction has far-reaching applications, from weather forecasting to sales prediction and marketing trends, directly impacting human life. For instance, precise weather forecasts can guide farmers in making informed agricultural decisions, enhancing crop yield and sustainability.

Regression, a fundamental task in machine learning, is dedicated to predicting continuous values based on input features. Numerous regression algorithms have been developed, including Linear Regression, Ridge Regression, Lasso Regression, and Support Vector Regression. These traditional methods are well-established but are often limited by their

reliance on strict mathematical assumptions or their sensitivity to data distribution. To explore beyond these limitations, researchers have continuously sought novel approaches to improve prediction performance.

One such innovative approach is Neurochaos Learning (NL) [1–3], a brain-inspired algorithm initially proposed for classification tasks. NL is characterized by transforming the original features of a dataset into chaotic features using chaotic maps, specifically the Skew Tent Map. These chaotic features—firing time, firing rate, energy, and entropy—are then used for classification by employing either cosine similarity or traditional machine learning algorithms. The success of NL in classification has inspired the exploration of its potential in regression tasks, an extension given that classification can be viewed as a specialized form of regression that predicts discrete labels instead of continuous values.

This study aims to extend the Neurochaos Learning framework to regression problems. The approach utilises a single chaotic feaure - Tracemean [4] and the original features of dataset which is then passed on to traditional regression algorithms including Linear Regression, Ridge Regression, Lasso Regression, and Support Vector Regression [5, 6]. By augmenting the original features with this single chaotic feature, we maintain the advantages of chaos-inspired transformations while significantly reducing computation time.

The proposed Neurochaos Learning-based regression algorithm is then evaluated using ten real life datasets : Diabetes [7], Bike Sharing [8], Fish [9], Life Expectancy [10], Concrete Strength [11], World Happiness Report [12], Body Fat [13], Auto MPG [14], Red Wine Quality [15], NASA Airfoil Self-Noise [16] and generated dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$. The performance is compared against existing classical methods using standard performance metrics such as R^2 score, Mean Squared Error (MSE), and Mean Absolute Error (MAE). This exploration not only demonstrates the versatility of NL but also highlights the potential of chaotic transformations in enhancing regression models, paving the way for more accurate and computationally efficient prediction frameworks.

2 Traditional Regression Algorithms

Traditional regression algorithms form the foundational methods in supervised learning used to model the relationship between input variables and a continuous output. These techniques are often effective and serve as a benchmark for evaluating newer models.

2.1 Linear Regression

Linear Regression [5] is one of the simplest and most widely used regression techniques. It models the relationship between a dependent variable and one or more independent variables by fitting a linear equation to the observed data.

Let $\{(y_i, x_{i1}, \ldots, x_{ik}), i = 1, 2, \ldots n\}$ be the dataset , where y is a continuous variable and $x_{i1}, \ldots, x_{ik}$ are continuous or appropriately coded categorical regressors. The regression model is given by

$$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \varepsilon_i, \quad i = 1, \ldots, n,$$

where the error terms $\varepsilon_1, \ldots, \varepsilon_n$ are assumed to be independent and identically distributed (i.i.d.) with mean zero and constant variance:

$$E(\varepsilon_i) = 0, \quad \text{Var}(\varepsilon_i) = \sigma^2.$$

The coefficients $\beta_0, \beta_1, \ldots, \beta_k$ are called as regression coefficients. The estimated regression function is given by:

$$\hat{f}(x_1, \ldots, x_k) = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_k x_k,$$

which serves as an estimator for the conditional expectation $E(y \mid x_1, \ldots, x_k)$. Thus, this function can be used for predicting the response variable y, commonly denoted by $\hat{y}$.

For the purposes of statistical inference, it is often further assumed that the error terms follow a normal distribution:

$$\varepsilon_i \sim \mathcal{N}(0, \sigma^2).$$

The estimation theory for regression coefficients in linear models is fundamentally based on the method of least squares(LS). While least squares is a widely used technique, other approaches such as maximum likelihood estimation can also be employed for parameter estimation.

According to the least squares principle, the regression coefficients $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)$ are estimated by minimizing the sum of squared residuals:

$$LS(\boldsymbol{\beta}) = \sum_{i=1}^{n} (y_i - \mathbf{x}_i^\top \boldsymbol{\beta})^2 = \sum_{i=1}^{n} \varepsilon_i^2 = \boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon},$$

where $\mathbf{x}_i$ is the vector of predictors for the i^{th} observation and ε_i is the residual error.

2.2 Ridge Regression

Ridge regression [5] addresses overfitting in linear regression by introducing a regularization technique that modifies the cost function with a penalty term. This penalty, proportional to the sum of the squared coefficients, constrains their magnitude. As the penalty strength increases, the coefficients are shrunk closer to zero, thereby reducing the model's variance and helping to generalize better on unseen data. The penalty term, calculated as the sum of squared coefficients, discourages large parameter values and leads to a modified objective function of the form:

$$\text{LS}(\boldsymbol{\beta}) = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) + \lambda \boldsymbol{\beta}^T \boldsymbol{\beta},$$

where λ controls the strength of regularization. By minimizing this penalized loss, ridge regression ensures a balance between fitting the data and maintaining model simplicity.

2.3 Lasso Regression

While ridge regression is effective for estimating regression coefficients in high-dimensional settings or when the features exhibits multicollinearity, it does not produce sparse solutions. That is, all regression coefficients are generally non-zero with probability one. For better model interpretability, it may be preferable not only to shrink coefficients but also to enforce exact zero values for some of them. This enables simultaneous model estimation and variable selection. Such an approach is achieved by replacing the squared coefficient penalty with a penalty on the absolute values of the coefficients, i.e.,

$$\text{pen}(\boldsymbol{\beta}) = \sum_{j=1}^{k} |\beta_j|.$$

This leads to the objective function for Lasso regression:

$$\text{LS}(\boldsymbol{\beta}) = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) + \lambda \sum_{j=1}^{k} |\beta_j|,$$

where λ controls the strength of regularization.

LASSO (Lasso) stands for Least Absolute Shrinkage and Selection Operator [5]. Ridge regression applies a quadratic penalty that heavily penalizes large coefficient values while exerting relatively little influence on those near zero . In contrast, the Lasso Regression uses an absolute value penalty that grows more slowly for large coefficients but increases more rapidly near zero. As a result, Lasso Regression tends to shrink smaller coefficients more aggressively toward zero, promoting sparsity, while having a reduced effect on larger coefficients.

2.4 Support Vector Regression

Support Vector Regression (SVR) [6] extends the principles of Support Vector Machines (SVM) to regression problems. Instead of minimizing the squared error as in traditional regression techniques, SVR aims to fit the best line within a specified threshold (called the ϵ-tube) such that the predictions deviate from the true targets by at most ϵ.

Given a training dataset $T = \{(x_i, y_i)\}_{i=1}^{l}$, where $x_i \in \mathbb{R}^N$ and $y_i \in \mathbb{R}$, Support Vector Regression aims to construct a linear function of the form:

$$f(x) = W^T \phi(x) + b, \tag{1}$$

where W is a vector in the feature space $\mathcal{F}$ and $\phi(x)$ denotes a mapping from the input space to the feature space.

The parameters W and b are determined by solving the following convex optimization problem:

$$\min_{W,b,\xi_i,\xi_i^*} \quad \frac{1}{2} W^T W + C \sum_{i=1}^{l} (\xi_i + \xi_i^*) \tag{2}$$

$$\text{subject to} \quad y_i - (W^T \phi(x_i) + b) \leq \epsilon + \xi_i \tag{3}$$

$$(W^T \phi(x_i) + b) - y_i \leq \epsilon + \xi_i^* \tag{4}$$

$$\xi_i, \xi_i^* \geq 0, \quad \text{for } i = 1, \ldots, l. \tag{5}$$

Here, the loss function penalizes only those predictions where the absolute deviation from the target value exceeds a user-specified threshold ϵ. The slack variables ξ_i and ξ_i^* represent the magnitude of positive and negative deviations respectively.

SVR is robust to outliers due to the ϵ-insensitive loss function. The model complexity is controlled by C and ϵ, which must be carefully tuned. SVR can be extended with kernels (e.g., RBF, polynomial) to handle non-linear regression problems.

Unlike Ridge and Lasso, SVR does not impose a penalty directly on the coefficient magnitudes, but rather on the amount of error exceeding a certain threshold. This makes SVR particularly useful for applications where small deviations are acceptable, and focus is placed on limiting large errors.

3 Proposed Algorithm

The proposed algorithm enhances traditional regression algorithms by incorporating both the original features and a chaotic feature—the arithmetic mean of the neural trace. This section outlines the key steps involved in the algorithm :

- *Step 1* : Normalize the dataset

  $$\{(x_1^1, x_2^1, \ldots, x_n^1), (x_1^2, x_2^2, \ldots, x_n^2), \ldots, (x_1^m, x_2^m, \ldots, x_n^m)\}$$

  using min-max normalization. Here, there are m samples with n features. After normalisation let the dataset be

  $$\{(z_1^1, z_2^1, \ldots, z_n^1), (z_1^2, z_2^2, \ldots, z_n^2), \ldots, (z_1^m, z_2^m, \ldots, z_n^m)\}$$

- *Step 2* : The input layer consists of n 1D chaotic neurons . Here each neuron is a skew tent map with skew value 0.499. Each neuron will start chaotic firing with an initial neural activity of q units and the neural trace stops when it reaches the epsilon ϵ neighbourhood of $z_1^i, z_2^i, \ldots, z_n^i$.

- *Step 3* : The only feature extracted from the chaotic neural trace corresponding to each input attribute x_i^j is mean of neural trace, Tracemean (TM), denoted by t_i^j. Unlike from NL algorithms for classification, here both the original features and transformed features are used for regression. Corresponding to the chaotic neural

firing of the neuron in the input layer of NL (i.e., for each z_i^j), arithmetic mean of its neural trace is computed. Hence if the input data is of size $m \times n$ then the new feature extracted data is also of the size $m \times 2n$. Let the extracted feature set be :

$$\{(f_1^1, f_2^1, \ldots, f_{2n}^1), (f_1^2, f_2^2, \ldots, f_{2n}^2), \ldots, (f_1^m, f_2^m, \ldots, f_{2n}^m\}.$$

where $f_i^j = z_i^j$ for $j = 1, 2, \ldots n$ and $f_i^j = t_i^j$ for $j = n+1, n+2, \ldots 2n$.

- *Step 4* : The extracted dataset, comprising input features and their corresponding mean of neural trace, can either be given as input to Linear Regression, Ridge Regression, Lasso Regression or Support Vector Regression. Accordingly, four distinct Neurochaos-based regression algorithms are proposed:
  - Augmented Chaotic Linear Regression(ACLR)
  - Augmented Chaotic Ridge Regression (ACRR)
  - Augmented Chaotic Lasso Regression (ACLR)
  - Augmented Chaotic Support Vector Regression (ACSVR)
- *Step 5* : Model will output a real valued predction for each sample.

The proposed algorithm is illustrated in figure 1.

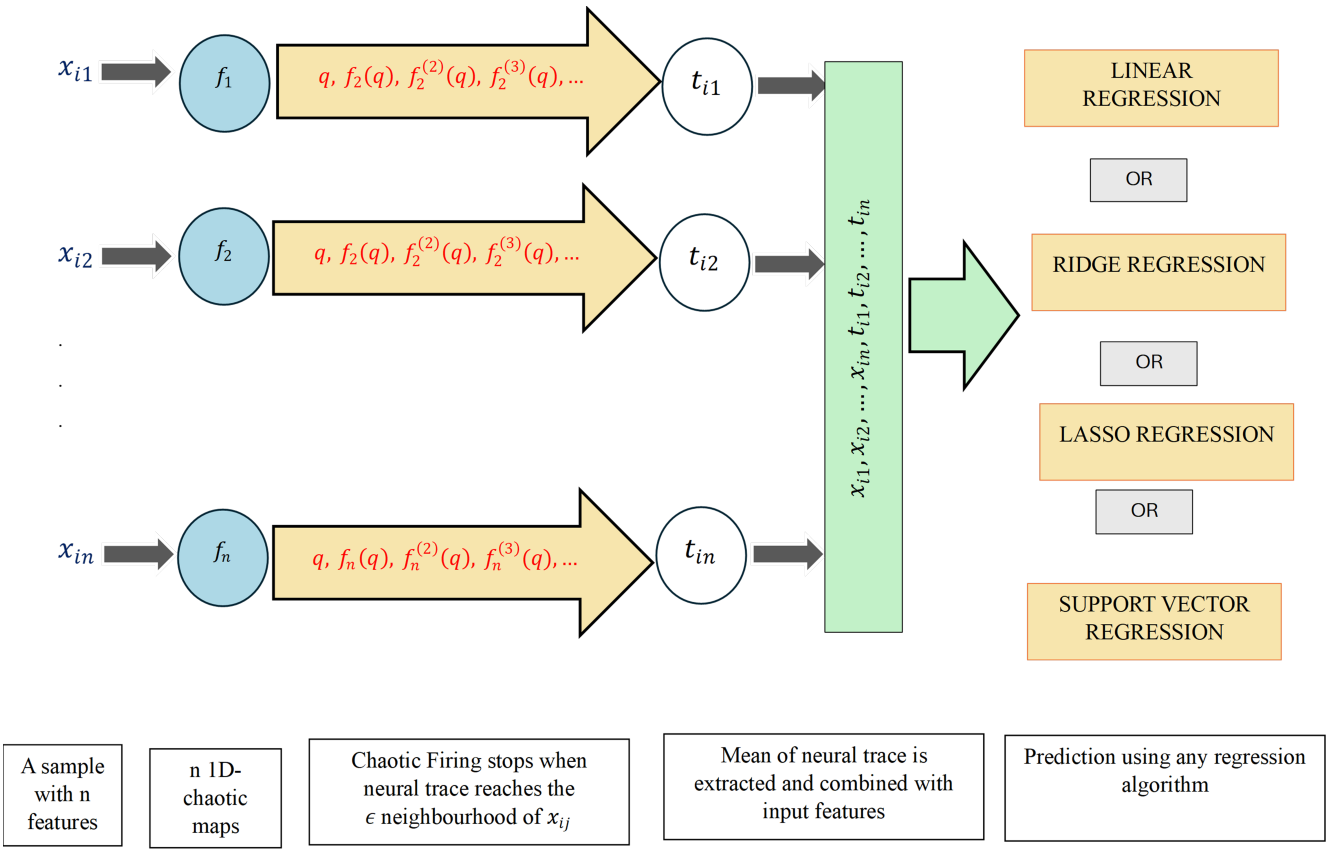


Figure 1: Augmented Chaotic Regression.

4 Results

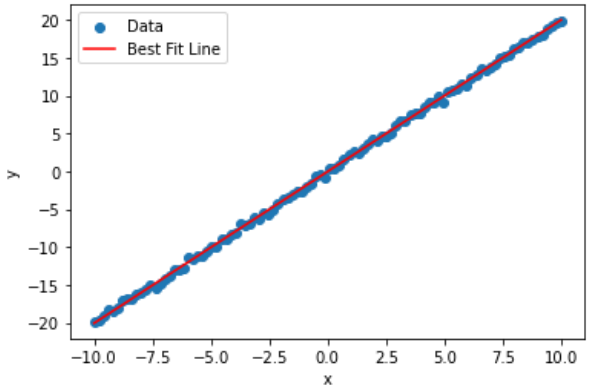
The performance of the proposed algorithms is analysed using two different ways :

- **Using ten different real life datasets** : The following datasets are used to analyse the performance of the algorithm: *Diabetes Dataset, Bike Sharing Dataset, Fish, Life Expectancy, Concrete Strength, World Happiness Report, Body Fat, Auto MPG, Red Wine Quality, NASA Airfoil Self-Noise*. The description of datasets is included in Appendix A.
- **Using the generated dataset(synthetic dataset)**: The dataset generated is of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$. The value of c is fixed as zero and m is taken as -2 and 2. The noise ϵ is randomly (Random state is fixed as 42 in our experiment) chosen from Normal distribution with mean zero and standard deviation varied among the values $5, 1$ and0.1. The value of x_i is taken from the interval $(-10, 10)$. Dataset of size $10, 50, 100$ and 1000 are generated for comparison. Thus we have considered 24 generated datasets for comparing the performance of proposed four augumented chaotic regression algorithms.

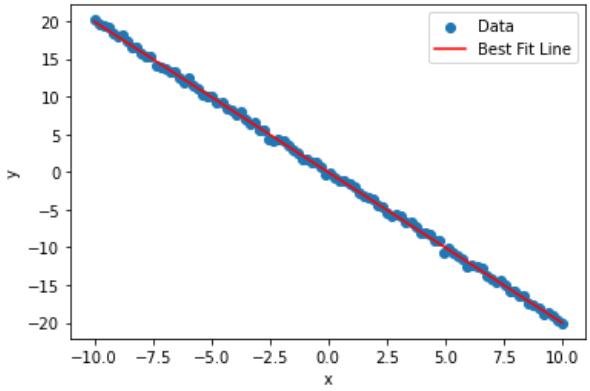
  1. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 10, \epsilon \in N(0, 5)\}$
  2. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 10, \epsilon \in N(0, 5)\}$
  3. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 10, \epsilon \in N(0, 1)\}$
  4. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 10, \epsilon \in N(0, 1)\}$
  5. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 10, \epsilon \in N(0, 0.1)\}$
  6. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 10, \epsilon \in N(0, 0.1)\}$
  7. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 50, \epsilon \in N(0, 5)\}$
  8. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 50, \epsilon \in N(0, 5)\}$
  9. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 50, \epsilon \in N(0, 1)\}$
  10. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 50, \epsilon \in N(0, 1)\}$
  11. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 50, \epsilon \in N(0, 0.1)\}$
  12. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 50, \epsilon \in N(0, 0.1)\}$
  13. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 100, \epsilon \in N(0, 5)\}$
  14. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 100, \epsilon \in N(0, 5)\}$
  15. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 100, \epsilon \in N(0, 1)\}$
  16. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 100, \epsilon \in N(0, 1)\}$
  17. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 100, \epsilon \in N(0, 0.1)\}$
  18. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 100, \epsilon \in N(0, 0.1)\}$
  19. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 1000, \epsilon \in N(0, 5)\}$
  20. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 1000, \epsilon \in N(0, 5)\}$
  21. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 1000, \epsilon \in N(0, 1)\}$
  22. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 1000, \epsilon \in N(0, 1)\}$
  23. $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, 1000, \epsilon \in N(0, 0.1)\}$
  24. $\{(x_i, y_i) : y_i = -2x_i + \epsilon, i = 1, 2, \ldots, 1000, \epsilon \in N(0, 0.1)\}$

  Dataset of size 100 for different values of m and ϵ is shown in Figure 2.

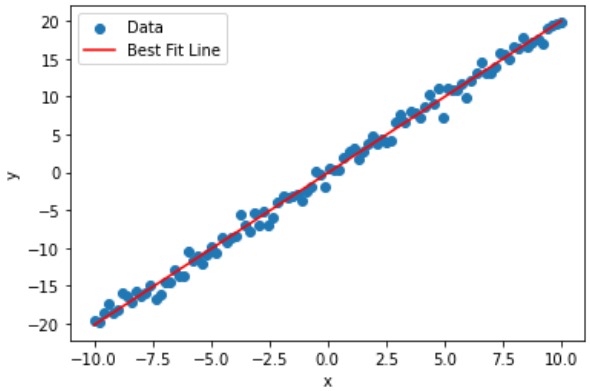
  Here the performance is compared with minimum mean square error(MMSE) corresponding to each dataset. MMSE is calculated by taking the difference between the predicted value and least square solution. Least square solution is computed using the pseudo-inverse method. The red line in Figure 2 represents the best fit line, the regression line that corresponds to MMSE.



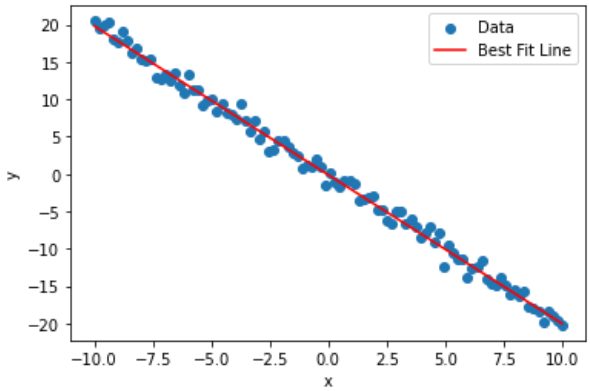
$m = 2, c = 0, \epsilon \in N(0, 0.1)$



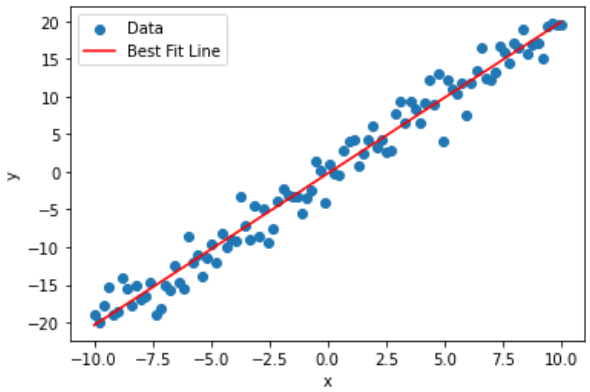
$m = -2, c = 0, \epsilon \in N(0, 0.1)$



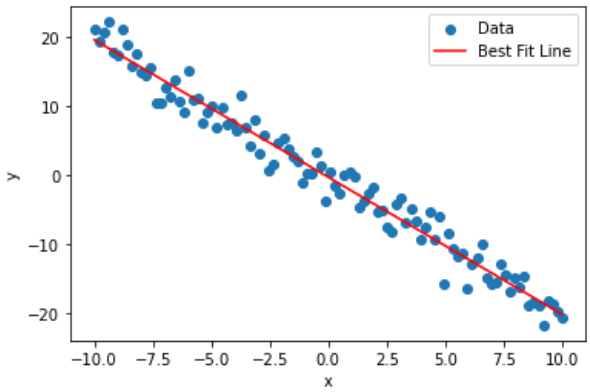
$m = 2, c = 0, \epsilon \in N(0, 1)$



$m = -2, c = 0, \epsilon \in N(0, 1)$



$m = 2, c = 0, \epsilon \in N(0, 5)$



$m = -2, c = 0, \epsilon \in N(0, 5)$

Figure 2: Dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, 100\}$.

4.1 Hyperparameter Tuning

Each dataset was initially split into training (80%) and testing (20%) sets. Training involves tuning of the hyperparameters associated with feature extraction - q and epsilon (Noise around Stimulus) and the regression algorithm (eg: Regularisation Parameter in SVR) using five fold cross validation. The Initial neural activity, q is tuned in the range 0.01 to 0.99 with a step value 0.01 and epsilon (Noise around Stimulus) is tuned in the range 0.01 to 0.45 with a step value 0.01.The regularisation parameter C in SVR is tuned among the values $1, 10, 50, 100$. The regularisation parameter α in Lasso Regression and Ridge Regression is tuned among the values $0.1, 1.0$ and 10.0. The maximum number of iterations used in Lasso Regression is 10000.

For real-world datasets, hyperparameter tuning was conducted with the objective of maximizing R^2 Score. The combination of hyperparameters that resulted in the highest R^2 Score was selected for model evaluation, as it was also observed to correspond to a low Mean Squared Error (MSE). Consequently, an independent tuning process based on MSE was deemed unnecessary. Following hyperparameter selection, model performance was assessed on a held-out test set comprising 20% of the data, using evaluation metrics including the R^2 Score, MSE, and Mean Absolute Error (MAE).

In the case of synthetically generated datasets, hyperparameters were optimized by minimizing the MSE. Accordingly, model performance on these datasets was evaluated primarily using the MSE metric.

The following section will present the value of hyperparameters and performance metrics for different datasets using different versions of augmented chaotic regression algorithms.

4.2 Augmented Chaotic Linear Regression

This section outlines the performance of the proposed algorithm, Augmented Chaotic Linear Regression, using both real world and generated datasets. This algorithm requires the tuning of only two hyperparameters q and epsilon (Noise around Stimulus). The table 1 presents the value of hyperparameters and the performance metrics R^2 Score, MSE and MAE for real world datasets. Tables 2, 3, 4, 5 correseponds to the dataset of size 10, 50, 100 and 1000 respectively

Table 1: **Hyperparameters and Performance metrics of Augmented Chaotic LR on real life datasets.**

Sl No	Datasets	q	epsilon	Training R^2 Score	Testing R^2 Score	MSE	MAE
1	Diabetes Dataset	0.11	0.21	0.505	0.271	3859.203	49.444
2	Bike Sharing Dataset	0.01	0.01	0.999	0.962	152936.267	372.184
3	Fish	0.03	0.44	0.952	0.875	17767.677	107.112
4	Life Expectancy	0.18	0.11	0.887	0.845	10.994	2.398
5	Concrete	0.65	0.08	0.802	0.776	64.148	5.97
6	Happiness	0.08	0.03	0.804	0.474	0.547	0.543
7	bodyfat	0.05	0.05	0.968	0.698	14.011	3.063
8	autompg	0.48	0.07	0.762	0.658	17.453	3.174
9	Wine quality	0.84	0.26	0.682	0.648	0.438	0.493
10	Airfoil noise	0.19	0.03	0.520	0.548	22.646	3.708

4.3 Augmented Chaotic Ridge Regression

This section outlines the performance of the proposed algorithm Augmented Chaotic Ridge Regression using both real world and generated datasets. This algorithm requires the tuning three hyperparameters q, epsilon (Noise around Stimulus) and α. The table 6 presents the value of hyperparameters and the performance metrics R^2 Score, MSE and MAE for real world datasets. Tables 7, 8, 9, 10 correseponds to the dataset of size 10, 50, 100 and 1000 respectively.

Table 2: **Hyperparameters and Performance metrics of Augmented Chaotic LR on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 10$.

Parameters to generate dataset	**q**	**epsilon**	**Training MSE**	**Testing MSE**
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.01	0.08	1.236	23.891
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.01	0.08	1.236	22.142
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.01	0.08	0.247	20.797
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.01	0.08	0.247	20.015
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.01	0.08	0.025	19.940
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.01	0.08	0.025	19.694

Table 3: **Hyperparameters and Performance metrics of Augmented Chaotic LR on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 50$**.**

Parameters to generate dataset	q	epsilon	Training MSE	Testing MSE
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.58	0.05	3.942	31.923
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.80	0.01	4.079	64.763
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.80	0.01	0.815	33.778
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.80	0.01	0.815	50.293
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.80	0.01	0.081	38.465
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.80	0.01	0.081	43.687

Table 4: **Hyperparameters and Performance metrics of Augmented Chaotic LR on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 100$**.**

Parameters to generate dataset	q	epsilon	Training MSE	Testing MSE
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.36	0.02	4.161	7.181
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.02	0.01	4.067	9.020
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.02	0.01	0.873	5.158
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.01	0.02	0.802	6.295
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.77	0.01	0.089	44.400
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.01	0.02	0.082	5.289

Table 5: **Hyperparameters and Performance metrics of Augmented Chaotic LR on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 1000$**.**

Parameters to generate dataset	q	epsilon	Training MSE	Testing MSE
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.02	0.01	4.795	4.795
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.01	0.01	4.835	4.847
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.01	0.01	0.959	0.985
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.01	0.01	0.974	1.018
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.01	0.01	0.098	0.139
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.01	0.01	0.103	0.150

Table 6: **Hyperparameters and Performance metrics of Augmented Chaotic RR on real life datasets.**

Sl no	Datasets	alpha	q	epsilon	Training R^2 Score	Testing R^2 Score	MSE	MAE
1	Diabetes Dataset	1	0.57	0.069	0.509	0.442	2953.824	42.774
2	Bike Sharing Dataset	0.1	0.01	0.01	0.999	0.962	151973.922	370.723
3	Fish	0.1	0.79	0.15	0.953	0.954	6516.313	67.618
4	Life Expectancy	0.1	0.18	0.11	0.885	0.846	10.925	2.401
5	Concrete	0.1	0.069	0.05	0.794	0.788	60.61	6.075
6	Happiness	0.1	0.03	0.06	0.809	0.509	0.51	0.531
7	Bodyfat	0.1	0.05	0.05	0.969	0.749	2.78	11.685
8	AutoMPG	0.1	0.48	0.069	0.763	0.658	17.441	3.187
9	Wine Quality	0.1	0.84	0.26	0.682	0.649	0.437	0.494
10	Airfoil Noise	0.1	0.19	0.03	0.52	0.548	22.622	3.707

Table 7: **Hyperparameters and Performance metrics of Augmented Chaotic RR on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 10$**.**

Parameters to generate dataset	alpha	q	epsilon	Training MSE	Testing MSE
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.08	0.13	10.127	17.227
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.01	0.069	8.596	7.844
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.08	0.13	7.652	10.590
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.08	0.13	7.188	5.732
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.08	0.13	7.001	8.111
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.08	0.13	6.854	6.575

Table 8: **Hyperparameters and Performance metrics of Augmented Chaotic RR on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 50$**.**

Parameters to generate dataset	alpha	q	epsilon	Training MSE	Testing MSE
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.32	0.03	4.455	30.842
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.06	0.01	4.364	58.846
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.65	0.01	1.067	32.829
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.8	0.01	1.019	47.270
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.26	0.01	0.263	35.873
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.8	0.01	0.254	41.095

Table 9: **Hyperparameters and Performance metrics of Augmented Chaotic RR on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 100$**.**

Parameters to generate dataset	alpha	q	epsilon	Training MSE	Testing MSE
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.02	0.01	4.253	6.633
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.02	0.01	4.104	8.763
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.02	0.01	0.915	4.882
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.02	0.01	0.849	5.835
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.3	0.01	0.133	4.687
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.02	0.01	0.130	4.983

Table 10: **Hyperparameters and Performance metrics of Augmented Chaotic RR on dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ with sample size $n = 1000$.**

Parameters to generate dataset	alpha	q	epsilon	Training MSE	Testing MSE
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.02	0.01	4.795	4.792
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.01	0.01	4.835	4.841
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.01	0.959	0.980
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.01	0.975	1.013
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.098	0.135
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.103	0.145

4.4 Augmented Chaotic Lasso Regression

This section outlines the performance of the proposed algorithm Augmented Chaotic Lasso Regression using both real world and generated datasets. This algorithm requires the tuning three hyperparameters q, epsilon (Noise around Stimulus) and α. The table 11 presents the value of hyperparameters and the performance metrics R^2 Score, MSE and MAE for real world datasets. Tables 12, 13, 14, 15 corresponds to the dataset of size 10, 50, 100 and 1000 respectively

Table 11: **Hyperparameters and Performance metrics of Augmented Chaotic LS on real life datasets.**

Sl No	Dataset	alpha	q	epsilon	Training R^2 Score	Testing R^2 Score	MSE	MAE
1	Diabetes	0.1	0.57	0.08	0.506	0.434	2998.53	43.137
2	Bike Sharing	0.1	0.01	0.01	0.999	0.961	153835.585	373.370
3	Fish	0.1	0.03	0.44	0.955	0.883	16597.001	104.213
4	Life Expectancy	0.1	0.18	0.11	0.852	0.829	12.111	2.463
5	Concrete	0.1	0.069	0.05	0.781	0.787	60.935	5.873
6	Happiness	0.1	0.35	0.26	0.677	0.577	0.440	0.539
7	Body Fat	0.1	0.18	0.01	0.966	0.748	11.681	2.576
8	Auto MPG	0.1	0.52	0.09	0.754	0.678	16.403	3.171
9	Wine Quality*	0.0001	0.84	0.26	0.682	0.651	0.434	0.491
10	Airfoil Noise	0.1	0.98	0.02	0.481	0.514	24.305	3.931

**Note:* Unlike the other datasets, the Wine quality dataset required tuning of α from the smaller values 0.0001, 0.001, and 0.01. Larger values such as 0.1, 1, and 10 resulted in an R^2 Score of 0 for standalone Lasso Regression.

Table 12: **Hyperparameters and Performance metrics of Augmented Chaotic LS on dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ with sample size $n = 10$.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.38	0.08	3.032	11.342
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.02	0.069	1.715	16.637
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.01	0.921	17.670
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.02	0.069	0.791	14.581
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.418	15.711
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.412	14.672

4.5 Augmented Chaotic SVR

This section outlines the performance of the proposed algorithm Augmented chaotic SVR Regression using both real world and generated datasets. This algorithm requires the tuning three hyperparameters q, epsilon (Noise around Stimulus) and C. For real world datasets, the kernel used is Radial Basis Function (RBF). But for generated data, the kernel used linear function. The table 16 presents the value of hyperparameters and the performance metrics R^2 Score,

Table 13: **Hyperparameters and Performance metrics of Augmented Chaotic LS on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 50$**.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.01	0.02	4.812	25.765
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.05	0.02	4.469	57.896
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.02	1.067	31.385
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.06	0.01	1.004	45.760
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.220	36.148
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.204	41.018

Table 14: **Hyperparameters and Performance metrics of Augmented Chaotic LS on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 100$**.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.13	0.01	4.254	8.102
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.01	0.06	4.297	8.540
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.3	0.01	1.006	4.774
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.06	0.957	5.602
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.3	0.01	0.207	4.451
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.02	0.204	4.700

MSE and MAE for real world datasets. Tables 17, 18, 19, 20 corresponds to the dataset of size 10, 50, 100 and 1000 respectively.

5 Comparative Analysis

To evaluate the effectiveness of the proposed augmented chaotic regression methods, a comparative analysis was conducted using four models: Linear Regression (LR) , Ridge Regression (RR), Lasso Regression (LS), and Support Vector Regression (SVR), along with their respective augmented counterparts. The R^2 scores across ten real-world datasets were compared to identify performance trends and insights.

The following key observations were made:

- Across almost all datasets, the augmented versions consistently outperform or match their base methods, demonstrating the potential of the augumentation using chaotic features. In particular, Support Vector Regression (SVR) and its augmented version exhibit the highest R^2 Scores across several datasets, indicating superior modeling capacity for nonlinear patterns.
- The Bike Sharing Dataset consistently achieves very high R^2 scores across all models. Similarly, the Fish Dataset yields high R^2 Score values for all methods, with the ACSVR achieving the highest (0.97).

Table 15: **Hyperparameters and Performance metrics of Augmented Chaotic LS on dataset of the form** $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ **with sample size** $n = 1000$**.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.01	0.03	4.903	4.755
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.01	0.01	4.950	4.888
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.01	1.075	1.012
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.01	1.094	1.059
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.217	0.173
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.223	0.188

Table 16: **Hyperparameters and Performance metrics of Augmented Chaotic SVR on real life datasets.**

Sl. No	Dataset	q	epsilon	c	Training R^2 Score	Testing R^2 score	MSE	MAE
1	Diabetes Dataset	0.05	0.099	50	0.478	0.471	2801.659	40.605
2	Bike Sharing Dataset	0.03	0.12	100	0.885	0.900	400155.243	455.279
3	Fish	0.069	0.29	100	0.902	0.970	4258.662	49.033
4	Life Expectancy	0.12	0.05	100	0.931	0.914	6.132	1.661
5	Concrete	0.5	0.01	100	0.910	0.854	41.860	4.690
6	Happiness	0.04	0.34	50	0.845	0.620	0.395	0.484
7	Bodyfat	0.03	0.12	50	0.975	0.829	7.960	2.133
8	Auto MPG	0.51	0.01	50	0.788	0.653	17.680	3.029
9	Wine Quality	0.89	0.01	10	0.754	0.718	0.350	0.438
10	Airfoil Noise	0.75	0.02	100	0.835	0.860	7.011	2.046

Table 17: **Hyperparameters and Performance metrics of Augmented Chaotic SVR on dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ with sample size $n = 10$.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.38	0.099	1.918	8.694
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.08	0.060	1.093	16.105
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.069	0.374	22.124
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.08	0.060	0.191	16.693
Size = 10, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.65	0.010	0.022	20.315
Size = 10, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.06	0.080	0.0198	19.152

Table 18: **Hyperparameters and Performance metrics of Augmented Chaotic SVR on dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ with sample size $n = 50$.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.8	0.01	4.164	27.708
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.06	0.01	4.198	57.440
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.35	0.01	0.879	56.960
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.8	0.01	0.845	50.239
Size = 50, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.8	0.01	0.0857	38.241
Size = 50, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.8	0.01	0.0840	43.828

Table 19: **Hyperparameters and Performance metrics of Augmented Chaotic SVR on dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ with sample size $n = 100$.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.36	0.02	4.135	8.101
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.02	0.01	3.989	8.795
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.04	0.01	0.874	5.137
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.02	0.01	0.798	5.820
Size = 100, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.77	0.01	0.089	43.896
Size = 100, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.02	0.084	5.346

Table 20: **Hyperparameters and Performance metrics of Augmented Chaotic SVR on dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ with sample size $n = 1000$.**

Dataset Parameters	alpha	q	epsilon	Training MSE	Testing MSE
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.02	0.01	4.799	4.778
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	0.01	0.01	4.829	4.827
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.01	0.958	0.989
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.01	0.01	0.973	1.014
Size = 1000, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.098	0.144
Size = 1000, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.01	0.01	0.102	0.149

- Algorithm with high R^2 score has better performance. Boost in R^2 Score is computed using

$$\text{Boost in } R^2 Score = \frac{R^2_{\text{Augumented Chaotic Regression}} - R^2_{Traditional Regression}}{R^2_{Traditional Regression}}.$$

  A dataset is considered "improved" if the boost value is positive. Average boost is calculated only over the improved datasets for each algorithm. The table 21 shows how many datasets show improvement for each of the four algorithms when using the augmented version. Along with the count, the average boost percentage across those improved datasets is provided. Tables 24, 29, 34, 39 presents performance metrics of Traditional regression algorithms.

- Moderate R^2 scores are observed for all models, indicating the dataset's complexity. Notably, Lasso Regression required careful tuning of the regularization parameter to avoid near-zero R^2 scores, highlighting its sensitivity.

Table 21: **Number of real life datasets with Improvement and Average Boost Percentage for Augumented Chaotic Regression Algorithms.**

Algorithm	No. of Datasets Improved	Average Boost (%)
ACLR	4	8.92%
ACLS	6	10.24%
ACRR	5	11.35%
ACSVR	6	4.95%

Corresponding bar graphs for each model comparison have been included to visually support the above analysis.

To compare the performance of the proposed algorithms, MSE and MMSE of generated datasets of different sizes are also compared.

- The figures 7 and 8 shows that MSE and MMSE corresponding to the dataset $\{(x_i, y_i) : y_i = 2x_i + \epsilon, i = 1, 2, \ldots, n, \epsilon \in N(0, 5)\}$ for $n = 10, 50, 100, 1000$ with LR, LS, RR and SVR and their augmented versions. It is clear that as sample size increases, MSE decreases and getting closer to MMSE.

- The Table 22 higlights the MSE that is more close to MMSE corresponding to the generated datasets of size 1000 under different noise levels.

- Lasso regression (LS) may produce MSE values lower than the MMSE for certain datasets (For example, for the generated dataset with $Size = 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$) due to the effect of the regularisation term, which can lead to underfitting or overfitting depending on the chosen regularisation parameter.Ridge (RR) typically doesn't show this behavior as much because it doesn't eliminate features, making it less likely to underfit or overfit as sharply as Lasso does with poor regularisation values.

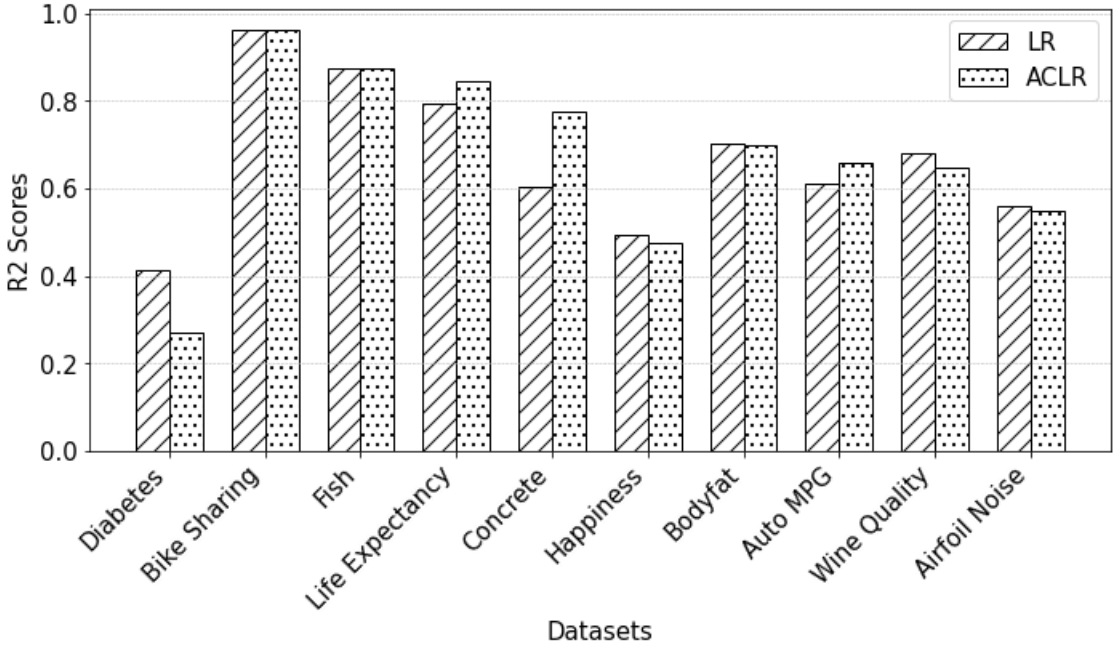


Figure 3: Comparison of R^2 Scores between LR and ACLR across Real-World Datasets.

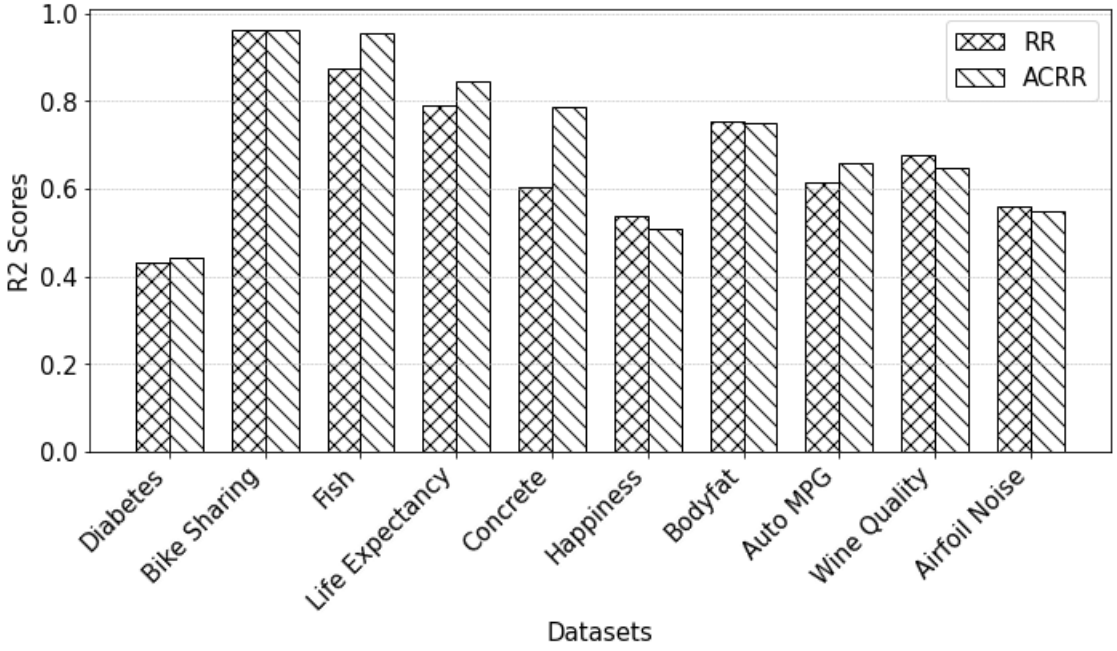


Figure 4: Comparison of R^2 Scores between RR and ACRR across Real-World Datasets.

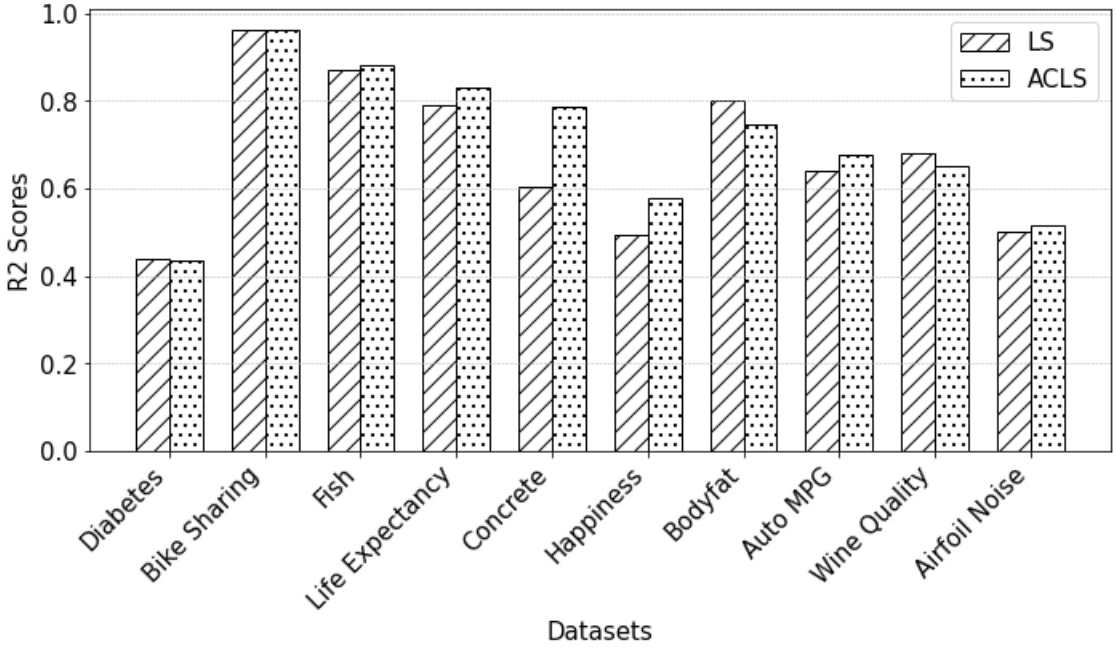


Figure 5: Comparison of R^2 Scores between LS and ACLS across Real-World Datasets.

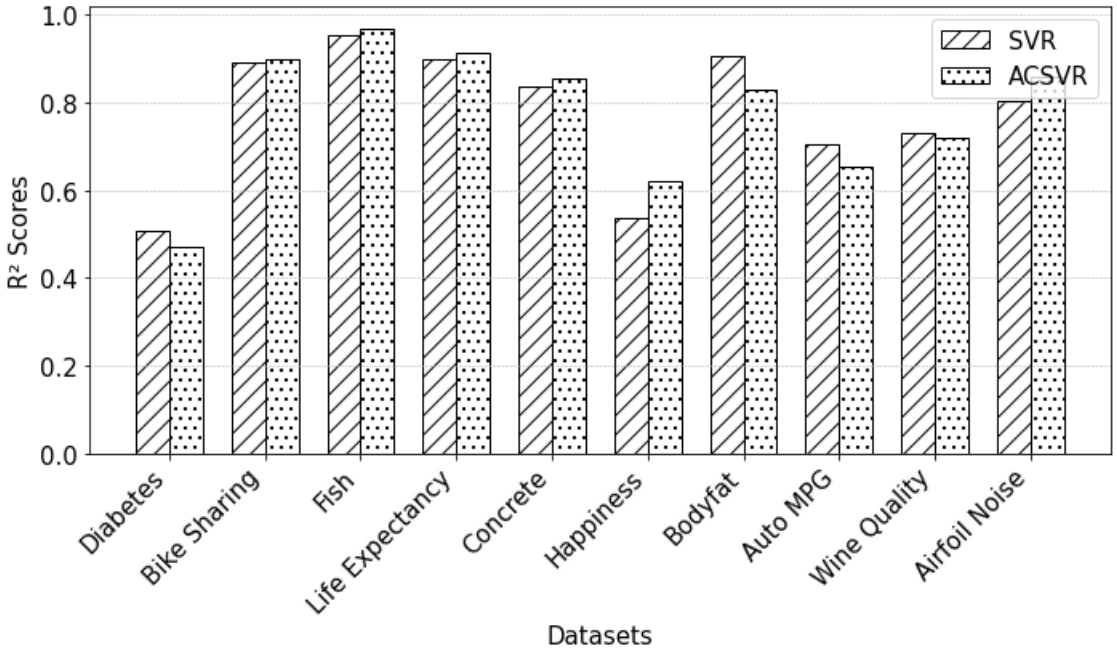


Figure 6: Comparison of R^2 Scores between SVR and ACSVR across Real-World Datasets.

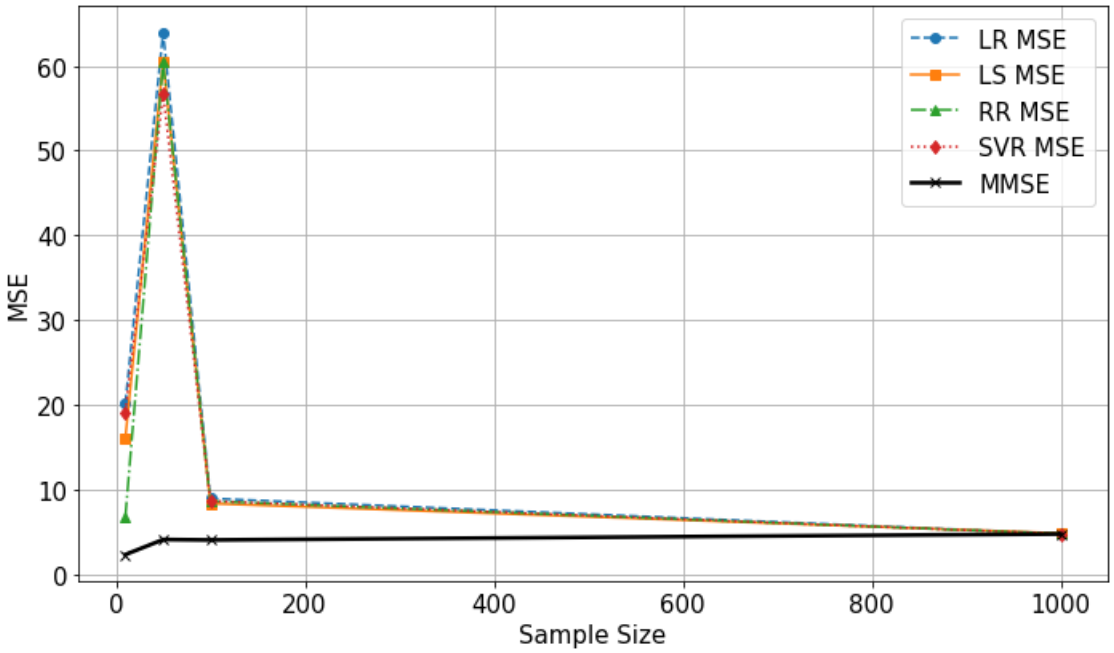


Figure 7: Comparison of MSE of traditional regression methods and MMSE corresponding to the generated datasets of different size.

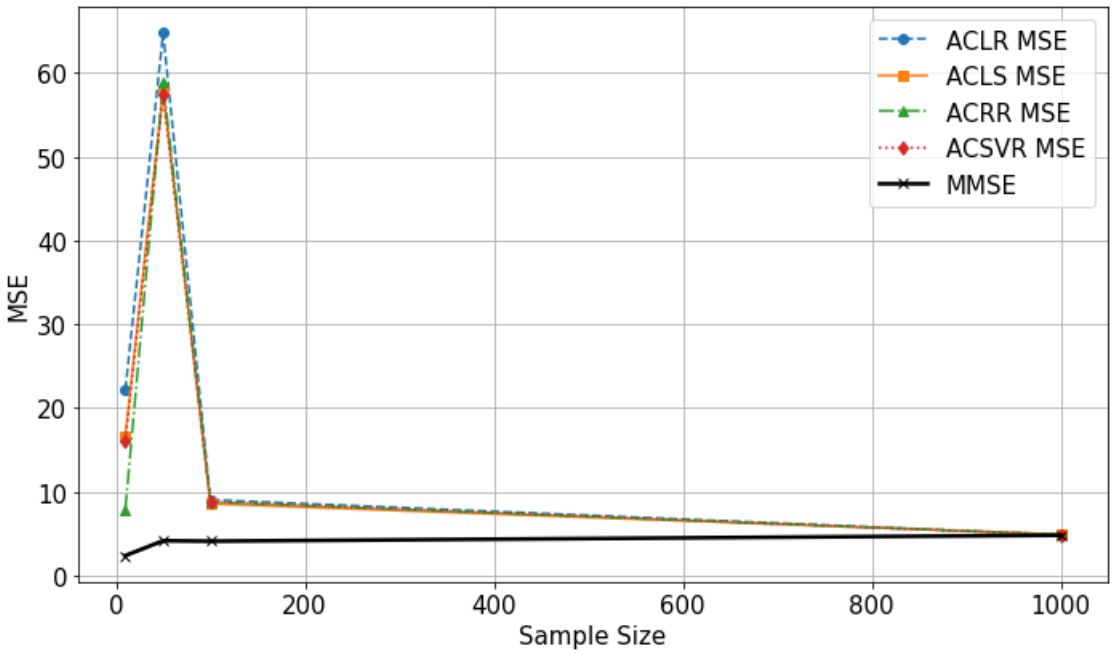


Figure 8: Comparison of MSE of Augmented chaotic regression methods and MMSE corresponding to the generated datasets of different size.

Table 22: **Comparison of MMSE and MSE values for different regression models on generated datasets of size** 1000**.**

Parameters to generate dataset	MMSE	SVR MSE	ASVR MSE	LR MSE	ACLR MSE	RR MSE	ACRR MSE	LS MSE	ACLS MSE
$Size = 1000, m = 2, c = 0, \epsilon \in N(0, 5)$	4.783	4.788	4.778	4.781	4.795	4.775	4.792	4.792	**4.755**
$Size = 1000, m = -2, c = 0, \epsilon \in N(0, 5)$	4.783	**4.806**	4.827	4.815	4.847	4.812	4.841	4.871	4.888
$Size = 1000, m = 2, c = 0, \epsilon \in N(0, 1)$	0.957	0.994	0.989	0.990	0.985	0.985	**0.980**	1.014	1.012
$Size = 1000, m = -2, c = 0, \epsilon \in N(0, 1)$	0.957	**1.001**	1.014	1.006	1.018	1.002	1.013	1.049	1.059
$Size = 1000, m = 2, c = 0, \epsilon \in N(0, 1)$	0.095	0.142	0.144	0.141	0.139	0.136	**0.135**	0.171	0.173
$Size = 1000, m = -2, c = 0, \epsilon \in N(0, 1)$	0.095	0.144	0.149	0.145	0.150	**0.141**	0.145	0.182	0.188

6 Conclusion

In this study, the performance of four augmented regression algorithms—Augmented Linear Regression, Augmented Ridge Regression, Augmented Lasso Regression, and Augmented Support Vector Regression (SVR)—was evaluated using ten different real-life datasets and a synthetically generated dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$. Our experimental analysis revealed that the inclusion of the chaotic Tracemean feature as an additional input significantly improved the performance of these regression models. Specifically, six out of the ten real-life datasets exhibited improved performance when the chaotic Tracemean feature was incorporated in both Augmented Lasso Regression and Augmented SVR, demonstrating the potential of chaos-inspired features in enhancing model accuracy.

Among the four augmented algorithms, Augmented Chaotic Ridge Regression exhibited the highest average performance boost, achieving an improvement of 11.35% compared to its non-augmented counterpart. Additionally, experiments conducted on the generated dataset showed that as the sample size increased, the Mean Squared Error (MSE) of all four augmented models consistently decreased and eventually converged towards the Minimum Mean Squared Error (MMSE), further validating the stability and scalability of the proposed approach.

Despite the promising results, this study is not without limitations. One of the primary challenges encountered was the need for extensive hyperparameter tuning, which can be time-consuming and may limit the practical applicability of the models in real-world scenarios.

Future work can focus on two key directions: first, exploring the inclusion of additional chaotic features to further enhance model performance, and second, developing efficient strategies to reduce training time, potentially through hyperparameter optimization techniques or parameter-free approaches.

Appendix

A Dataset Description

Table 23: **Dataset Details for Regression Analysis.**

Sl. No	Dataset	Number of Samples	Number of Features
1	Diabetes Dataset	442	10
2	Bike Sharing Dataset	731	13
3	Fish	159	5
4	Life Expectancy	1649	18
5	Concrete Strength	1030	8
6	World Happiness Report	156	6
7	Body Fat	252	14
8	Auto MPG	392	5
9	Red Wine Quality	1599	10
10	NASA Airfoil Self-Noise	1503	5

A.1 Diabetes Dataset

The Scikit-learn Diabetes Dataset (also known as the Sklearn Diabetes dataset) [7] includes ten features , such as age, sex, body mass index (BMI), average blood pressure, and six blood serum measurements, collected from 442 diabetes patients. The target variable represents a quantitative assessment of disease progression one year after the baseline measurements.

A.2 Bike Sharing Dataset

Bike Sharing dataset [8] contains the hourly and daily count of rental bikes from 2011 to 2012 in Capital bikeshare system with the corresponding weather and seasonal information.

A.3 Fish Dataset

The fish market dataset [9] is a collection of data related to various species of fish and their characteristics. The dataset is structured in such a way that each row corresponds to a single fish with its species and various physical measurements (lengths, height, and width). The target variable represents the weight of a fish based on its species and the provided physical measurements.

A.4 Life Expectancy Dataset

The Life Expectancy dataset [10] consists of 22 Columns and 2938 rows , including 20 predictive variables. These variables are divided into several broad categories: Immunization related factors, Mortality factors, Economical factors and Social factors. The target variable represents the life expectancy of people in various countries of the world.

A.5 Concrete Strength Dataset

The Concrete Dataset [11] provides essential information on concrete mixtures, including ingredient composition and strength characteristics.The target variable represents the compressive strength of the concrete.

A.6 World Happiness Report

The Happiness Dataset [12] includes a happiness score and six key contributing factors: economic production, social support, life expectancy, freedom, absence of corruption, and generosity. The happiness score estimates the extent to which each of these six factors contributes to making life evaluations higher in each country than they are in Dystopia, a hypothetical country that has values equal to the world's lowest national averages for each of the six factors. The target variable represents happiness score.

A.7 Body Fat Dataset

The Body Fat Dataset [13] includes measurement of body fat, age, weight, height, neck circumference, Knee circumference, ankle circumference, etc. The target variable represents the measurement of body fat.

A.8 Auto MPG Dataset

The Auto MPG Dataset [14] is on city-cycle fuel consumption measured in miles per gallon (mpg). The dataset includes Car name, model year, weight, cylinders, etc. The target variable represents Mileage per gallon performances of various cars.

A.9 Red Wine Quality Dataset

The Wine Quality Dataset [15] contains physicochemical properties of Portuguese "Vinho Verde" red wines and their corresponding quality scores, ranging from 0 to 10. It includes 11 input features such as acidity, alcohol content, pH, sulphates, and sugar levels, which influence the sensory evaluation of wine. The target variable represents the quality of wine.

A.10 NASA Airfoil Self-Noise Dataset

The NASA Airfoil Self-Noise Dataset [16] contains aerodynamic and acoustic test data collected from NACA 0012 airfoil blade sections in an anechoic wind tunnel. It includes five input features—frequency, angle of attack, chord length, free-stream velocity, and suction side displacement thickness—used to predict the scaled sound pressure level (SSPL) in decibels.

B Performance Analysis of traditional regression methods

This section presents the value of performance metrics of traditional regression methods including Linear Regression, Ridge Regression, Lasso Regression and Support Vector Regression. The performance is analysed on ten distict real life datasets described in section A and on the generated dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$.

B.1 Linear Regression(LR)

Linear Regression does not involve any tunable hyperparameters. The table 24 presents the value of hyperparameters and the performance metrics R^2 Score, MSE and MAE for real world datasets. Tables 25, 26, 27, 28 corresponds to the dataset of size 10, 50, 100 and 1000 respectively

B.2 Ridge Regression(RR)

The regularisation parameter α in Ridge Regression is tuned among the values $0.1, 1.0$ and 10.0. The table 29 presents the value of hyperparameters and the performance metrics R^2 Score, MSE and MAE for real world datasets. Tables 30, 31, 32, 33 corresponds to the dataset of size 10, 50, 100 and 1000 respectively

Table 24: **MSE of LR on real life datasets.**

Sl. No	Dataset	R^2 Score	MSE	MAE
1	Diabetes	0.413	3109.163	43.485
2	Bike Sharing	0.961	154179.697	373.843
3	Fish	0.875	17662.023	102.336
4	Life Expectancy	0.792	14.762	2.904
5	Concrete	0.604	113.229	8.426
6	Happiness	0.492	0.528	0.529
7	Bodyfat	0.704	13.773	2.970
8	Auto MPG	0.611	19.859	3.740
9	Wine Quality	0.679	0.400	0.494
10	Airfoil Noise	0.558	22.128	3.672

Table 25: **MSE of LR on generated dataset of size 10.**

Parameters to generate dataset	LR MSE
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	20.014
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	20.192
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	22.560
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	18.802
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	20.440
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	19.251

Table 26: **MSE of LR on generated dataset of size 50.**

Parameters to generate dataset	LR MSE
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	27.524
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	63.974
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	33.775
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	50.076
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	38.487
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	43.643

Table 27: **MSE of LR on generated dataset of size 100.**

Parameters to generate dataset	LR MSE
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	7.321
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	8.972
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	5.294
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	6.033
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	4.988
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	5.221

Table 28: **MSE of LR on generated dataset of size 1000.**

Parameters to generate dataset	LR MSE
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	4.781
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	4.815
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.990
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	1.006
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.141
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.145

Table 29: **MSE of RR on real life datasets.**

Sl. No.	Dataset	α	Training R^2 Score	Testing R^2 Score	MSE	MAE
1	Diabetes Dataset	0.1	0.480	0.430	3022.264	43.083
2	Bike Sharing Dataset	0.1	0.999	0.962	153219.576	372.521
3	Fish	0.1	0.874	0.874	17943.463	105.318
4	Life Expectancy	0.1	0.826	0.789	14.969	2.929
5	Concrete	0.1	0.602	0.605	112.990	8.431
6	Happiness	1.0	0.784	0.538	0.480	0.517
7	Bodyfat	0.1	0.967	0.752	11.518	2.691
8	AutoMPG	0.1	0.706	0.613	19.744	3.722
9	Wine Quality	0.1	0.647	0.678	0.401	0.497
10	Airfoil Noise	0.1	0.492	0.558	22.142	3.675

Table 30: **MSE of RR on generated dataset of size 10.**

Parameters to generate dataset	Training MSE	Testing MSE
Size $= 10$, m = 2, c = 0, $\epsilon \in N(0, 5)$	12.240	10.438
Size $= 10$, m = -2, c = 0, $\epsilon \in N(0, 5)$	11.533	6.768
Size $= 10$, m = 2, c = 0, $\epsilon \in N(0, 1)$	9.885	5.720
Size $= 10$, m = -2, c = 0, $\epsilon \in N(0, 1)$	9.568	4.079
Size $= 10$, m = 2, c = 0, $\epsilon \in N(0, 0.1)$	9.291	4.326
Size $= 10$, m = -2, c = 0, $\epsilon \in N(0, 0.1)$	9.191	3.807

Table 31: **MSE of RR on generated dataset of size 50.**

Parameters to generate dataset $y = mx + c +$ Noise	Training MSE	Testing MSE
Size $= 50$, m = 2, c = 0, $\epsilon \in N(0, 5)$	4.899	26.144
Size $= 50$, m = -2, c = 0, $\epsilon \in N(0, 5)$	4.800	60.425
Size $= 50$, m = 2, c = 0, $\epsilon \in N(0, 1)$	1.131	31.831
Size $= 50$, m = -2, c = 0, $\epsilon \in N(0, 1)$	1.086	47.162
Size $= 50$, m = 2, c = 0, $\epsilon \in N(0, 0.1)$	0.273	36.220
Size $= 50$, m = -2, c = 0, $\epsilon \in N(0, 0.1)$	0.259	41.068

Table 32: **MSE of RR on generated dataset of size 100.**

Parameters to generate dataset	Training MSE	Testing MSE
Size $= 100$, m = 2, c = 0, $\epsilon \in N(0, 5)$	4.497	7.047
Size $= 100$, m = -2, c = 0, $\epsilon \in N(0, 5)$	4.513	8.649
Size $= 100$, m = 2, c = 0, $\epsilon \in N(0, 1)$	0.932	5.006
Size $= 100$, m = -2, c = 0, $\epsilon \in N(0, 1)$	0.940	5.723
Size $= 100$, m = 2, c = 0, $\epsilon \in N(0, 0.1)$	0.132	4.692
Size $= 100$, m = -2, c = 0, $\epsilon \in N(0, 0.1)$	0.134	4.919

Table 33: **MSE of RR on generated dataset of size 1000.**

Parameters to generate dataset	Training MSE	Testing MSE
Size $= 1000$, m = 2, c = 0, $\epsilon \in N(0, 5)$	4.805	4.775
Size $= 1000$, m = -2, c = 0, $\epsilon \in N(0, 5)$	4.805	4.812
Size $= 1000$, m = 2, c = 0, $\epsilon \in N(0, 1)$	0.961	0.985
Size $= 1000$, m = -2, c = 0, $\epsilon \in N(0, 1)$	0.961	1.002
Size $= 1000$, m = 2, c = 0, $\epsilon \in N(0, 0.1)$	0.096	0.136
Size $= 1000$, m = -2, c = 0, $\epsilon \in N(0, 0.1)$	0.096	0.141

B.3 Lasso Regression(LS)

The regularisation parameter α in Lasso Regression and Ridge Regression is tuned among the values $0.1, 1.0$ and 10.0. The maximum number of iterations used in Lasso Regression is 10000. The table 34 presents the value of hyperparameters and the performance metrics R^2 Score, MSE and MAE for real world datasets. Tables 35, 36, 37, 38 correseponds to the dataset of size 10, 50, 100 and 1000 respectively

Table 34: **MSE of LS on real life datasets.**

Sl. No.	Dataset	α	Training R^2 Score	Testing R^2 Score	MSE	MAE
1	Diabetes Dataset	0.1	0.481	0.439	2968.989	42.904
2	Bike Sharing Dataset	0.1	0.999	0.961	154087.428	373.799
3	Fish	0.1	0.875	0.872	18104.369	104.990
4	Life Expectancy	0.1	0.805	0.790	14.899	2.875
5	Concrete	0.1	0.596	0.603	113.502	8.471
6	Happiness	0.1	0.534	0.495	0.525	0.565
7	Bodyfat	0.1	0.965	0.802	9.207	2.444
8	Auto MPG	0.1	0.704	0.641	18.306	3.494
9	Wine Quality *	0.0001	0.647	0.680	0.397	0.493
10	Airfoil Noise	0.1	0.461	0.502	24.910	3.999

**Note:* Unlike the other datasets, the Wine quality dataset required tuning of α from the smaller values 0.0001, 0.001, and 0.01. Larger values such as 0.1, 1, and 10 resulted in an R^2 Score of 0 for standalone Lasso Regression.

Table 35: **MSE of LS on generated dataset of size 10.**

Parameters to generate dataset	α	Training MSE	Testing MSE
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	3.139	23.413
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	3.096	16.066
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.921	17.670
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	0.901	14.384
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.418	15.711
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.412	14.672

Table 36: **MSE of LS on generated dataset of size 50.**

Parameters to generate dataset	α	Training MSE	Testing MSE
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	4.858	26.046
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	4.750	60.487
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	1.079	31.742
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	1.031	47.145
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.220	36.148
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.204	41.018

B.4 Support Vector Regression (SVR)

The regularisation parameter C in SVR is tuned among the values $1, 10, 50, 100$. The kernel used in real life datasets is radial basis function and is linear in generated dataset. The table 39 presents the value of hyperparameters and the performance metrics R^2 Score, MSE and MAE for real world datasets. Tables 40, 41, 42, 43 correseponds to the dataset of size 10, 50, 100 and 1000 respectively

Table 37: **MSE of LS on generated dataset of size 100.**

Parameters to generate dataset	α	Training MSE	Testing MSE
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	4.568	6.831
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	4.597	8.374
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	1.006	4.774
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	1.018	5.464
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.207	4.451
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.211	4.669

Table 38: **MSE of LS on generated dataset of size 1000.**

Parameters to generate dataset	α	Training MSE	Testing MSE
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	4.923	4.792
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	0.1	4.926	4.871
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	1.080	1.014
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	0.1	1.081	1.049
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.216	0.171
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	0.1	0.216	0.182

Table 39: **MSE of SVR on real life datasets.**

Sl no	Datasets	C - standalone	Testing R^2 Score	MSE	MAE
1	Diabetes Dataset	50	0.508	2602.347	40.55
2	Bike Sharing Dataset	100	0.890	440614.581	475.00
3	Fish	100	0.952	6824.104	46.58
4	Life Expectancy	100	0.900	7.097	1.702
5	Concrete	100	0.836	46.818	4.859
6	Happiness	10	0.535	0.483	0.574
7	Bodyfat	50	0.907	4.325	1.702
8	AutoMPG	10	0.704	15.107	2.832
9	Wine Quality	10	0.731	0.334	0.436
10	Airfoil Noise	100	0.803	9.840	2.253

Table 40: **MSE of SVR on generated dataset of size 10.**

Parameters to generate dataset	C value	Training MSE	Testing MSE
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	100	6.629	27.026
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	100	3.978	19.169
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	100	1.208	22.317
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	100	0.769	18.228
Size $= 10$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	100	0.096	19.643
Size $= 10$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	100	0.079	18.427

Table 41: **MSE of SVR on generated dataset of size 50.**

Parameters to generate dataset	C value	Training MSE	Testing MSE
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	50	5.018	31.180
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	50	4.868	56.703
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	100	0.992	35.655
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	100	0.965	47.614
Size $= 50$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	100	0.097	38.740
Size $= 50$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	100	0.0951	43.399

Table 42: **MSE of SVR on generated dataset of size 100.**

Parameters to generate dataset	C value	Training MSE	Testing MSE
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	50	4.489	7.389
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	100	4.474	8.666
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	50	0.896	5.264
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	100	0.895	5.855
Size $= 100$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	50	0.088	5.033
Size $= 100$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	50	0.088	5.142

Table 43: **MSE of SVR on generated dataset of size 1000.**

Parameters to generate dataset	C value	Training MSE	Testing MSE
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 5)$	100	4.811	4.788
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 5)$	50	4.812	4.806
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 1)$	100	0.960	0.994
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 1)$	100	0.960	1.001
Size $= 1000$, $m = 2$, $c = 0$, $\epsilon \in N(0, 0.1)$	100	0.096	0.142
Size $= 1000$, $m = -2$, $c = 0$, $\epsilon \in N(0, 0.1)$	50	0.096	0.144

C Minimum Mean Square Error(MMSE) of generated dataset

The tables 44, 45, 46, 47 corresponds to the MMSE of the dataset of the form $\{(x_i, y_i) : y_i = mx_i + c + \epsilon, i = 1, 2, \ldots, n\}$ for $n = 10$, $n = 50$, $n = 100$, $n = 1000$ respectively. Here ϵ denotes noise and is randomly chosen from a normal distribution with mean zero.

Table 44: **MMSE on generated dataset of size 10.**

n	m	c	Noise Distribution	MMSE
10	2	0	$N(0, 5)$	2.35
10	-2	0	$N(0, 5)$	2.35
10	2	0	$N(0, 1)$	0.47
10	-2	0	$N(0, 1)$	0.47
10	2	0	$N(0, 0.1)$	0.047
10	-2	0	$N(0, 0.1)$	0.047

Table 45: **MMSE on generated dataset of size 50.**

n	m	c	Noise Distribution	MMSE
50	2	0	$N(0,5)$	4.125
50	-2	0	$N(0,5)$	4.125
50	2	0	$N(0,1)$	0.825
50	-2	0	$N(0,1)$	0.825
50	2	0	$N(0,0.1)$	0.0825
50	-2	0	$N(0,0.1)$	0.0825

Table 46: **MMSE on generated dataset of size 100.**

n	m	c	Noise Distribution	MMSE
100	2	0	$N(0,5)$	4.074
100	-2	0	$N(0,5)$	4.074
100	2	0	$N(0,1)$	0.815
100	-2	0	$N(0,1)$	0.815
100	2	0	$N(0,0.1)$	0.081
100	-2	0	$N(0,0.1)$	0.081

Table 47: **MMSE on generated dataset of size 1000.**

n	m	c	Noise Distribution	MMSE
1000	2	0	$N(0,5)$	4.783
1000	-2	0	$N(0,5)$	4.783
1000	2	0	$N(0,1)$	0.957
1000	-2	0	$N(0,1)$	0.957
1000	2	0	$N(0,0.1)$	0.095
1000	-2	0	$N(0,0.1)$	0.095

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