Quantum Mechanics of Open Systems in Non-Inertial Motion

Zi-Fan Zhu, Yu Su,* Yao Wang,† Rui-Xue Xu,‡ and YiJing Yan
Hefei National Research Center for Physical Sciences at the Microscale and Department of Chemical Physics, University of Science and Technology of China, Hefei, Anhui 230026, China and
Hefei National Laboratory, University of Science and Technology of China, Heifei 230088, China
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The study of quantum mechanics in non-inertial reference frames, particularly in the context of open systems, introduces several intriguing phenomena and challenges. This paper presents a comprehensive framework for analyzing the quantum mechanics of open systems undergoing non-inertial motion. Our methodology leverages the concept of dissipatons, statistical quasi-particles that capture collective dissipative effects from the environment. We demonstrate that our approach offers a natural understanding of the intricate dynamics among non-inertial effects, decoherence, dissipation, and system-bath entanglement. Specifically, we conduct demonstrations focusing on the Lamb shift phenomenon within a rotating ring cavity. Through theoretical exposition and practical applications, our framework elucidates the profound interplay between open quantum dynamics and non-inertial motion, paving the way for advancements in quantum information processing and sensing technologies.

I. INTRODUCTION

In quantum physics, exploring open quantum systems has become a captivating research frontier [1–5]. Open systems, which interact with their surrounding environment, exhibit diverse phenomena spanning quantum optics [6–10], nuclear magnetic resonance [11–13], condensed matter [14–18], quark-gluon plasma [19–22], nonlinear spectroscopy [23–30], and chemical and biological physics [31–36]. The interaction between open quantum system and its environment often induces decoherence, a process in which the system's quantum coherence diminishes gradually, leading to classical-like behavior. Decoherence stems from the unavoidable entanglement with the environment, causing a rapid loss of delicate quantum superposition states. Consequently, the system is driven towards a mixed state, making it more susceptible to classical statistical treatments. In open quantum systems, decoherence plays a vital role in understanding the fundamental limits of quantum technologies and the boundary between classical and quantum behaviors [37].

Non-inertial effects, arising from the acceleration or rotation of the system, introduce novel dynamics and unique features that set them apart from their inertial counterparts [38]. Non-inertial effects complicate the dynamics of open quantum systems, including the quantum decoherence. Such effects have garnered significant attention in recent years, primarily due to their potential applications in various fields, including quantum information processing, precision measurements, and quantum metrology [39–42].

Furthermore, non-inertial effects have also been investigated in the context of quantum entanglement dynamics [43, 44]. Entanglement describes the correlation between two or more distant quantum systems. The acceleration or rotation of an open quantum system can lead to remarkable changes in the entanglement structure between the system and its environment. Understanding these dynamics is crucial in various scenarios such as quantum teleportation, quantum cryptography, and quantum communication protocols [45–47].

In this paper, we aim to provide a comprehensive framework for analyzing the quantum mechanics of open systems undergoing non-inertial motion. In Sec. II and III, we will delve into the mathematical formalism necessary to describe the non-inertial effects in open quantum systems, followed by a comprehensive framework, leverages the concept of dissipatons, statistical quasi-particles that capture collective dissipative effects from the environment [48, 49]. Finally, we provide a concise summary of this paper and deliver a preview of our future research endeavors. Throughout this paper, we set $\hbar = c = 1$ and $\beta = 1/(k_B T)$, with k_B being the Boltzmann constant and T the temperature. Besides, we denote the space–time coordinate as $x^\mu = (t, \boldsymbol{r}) = (t, r^i)$ with $\mu \in \{0, 1, 2, 3\}$ and $i \in \{1, 2, 3\}$ and the Minkowski metric as $\eta^{\mu\nu} = \mathrm{diag}\{-1, 1, 1, 1\}$. The Einstein summation convention is also adopted for the space–time indices.

II. NON-INERTIAL QUANTUM MECHANICS

A. Non-inertial unitary transformation

In this section, we briefly review the non-inertial quantum mechanics [50]. Let us start with the Hamiltonian,

$$H(\hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}) = \frac{[\hat{\boldsymbol{p}} - e\boldsymbol{A}(\hat{\boldsymbol{r}}, t)]^2}{2m} + V(\hat{\boldsymbol{r}}, t) + e\phi(\hat{\boldsymbol{r}}, t), \tag{1}$$

where m and e are the mass and charge of the particle, respectively. We use the hat accent ˆ to distinguish operators and c-numbers for $\boldsymbol{r}$ and $\boldsymbol{p}$. Here, $\boldsymbol{A}(\hat{\boldsymbol{r}}, t)$ and $\phi(\hat{\boldsymbol{r}}, t)$

* suyupilemao@mail.ustc.edu.cn
† wy2010@ustc.edu.cn
‡ rxxu@ustc.edu.cn

are the electromagnetic (EM) potentials, while $V(\hat{\boldsymbol{r}},t)$ is the potential energy. All of them possibly contain non-inertial effects as elaborated below.

To be concrete, we set

$$V(\hat{\boldsymbol{r}},t) = V_0\big(\hat{\boldsymbol{r}}'(t)\big), \tag{2}$$

with $\hat{\boldsymbol{r}}' = \mathsf{R}_t^{-1}(\hat{\boldsymbol{r}} - \boldsymbol{\zeta}_t)$, and V_0 being a specific potential in the body-fixed coordinate system. Here, $\mathsf{R}_t \in \mathrm{SO}(3)$ and $\boldsymbol{\zeta}_t \in \mathbb{R}^3$. By convention, we set the initial conditions: $\mathsf{R}_0 = \mathsf{I}$, and $\boldsymbol{\zeta}_0 = 0$; See Fig. 1 for the illustration. Special

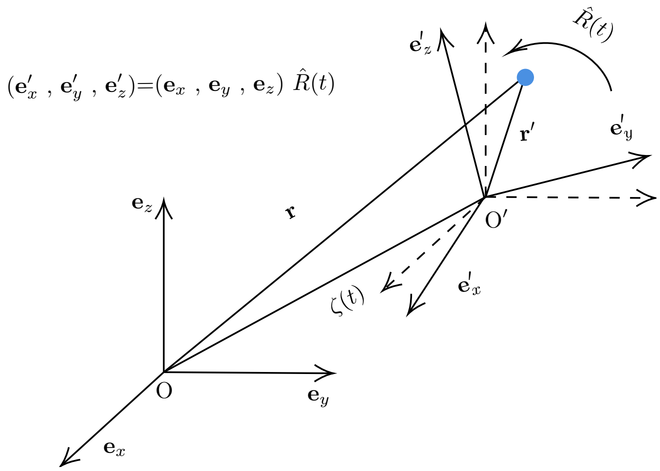

FIG. 1. Coordinate transformation of the mixed accelerating and rotational motions.

cases include:

1. Pure acceleration: $\mathsf{R}_t \equiv \mathsf{I}$, $V(\hat{\boldsymbol{r}},t) = V_0(\hat{\boldsymbol{r}} - \boldsymbol{\zeta}_t)$. Especially, $\boldsymbol{\zeta}_t = \boldsymbol{v}t$ and $\boldsymbol{\zeta}_t = (1/2)\boldsymbol{a}t^2$ correspond to the boost and the constant acceleration, respectively;

2. Pure rotation: $\boldsymbol{\zeta}_t \equiv 0$, $V(\hat{\boldsymbol{r}},t) = V_0(\mathsf{R}_t^{-1}\hat{\boldsymbol{r}})$. Generally, R_t can be expressed as,

$$\exp_+\left[\int_0^t \mathrm{d}\tau\ \Omega(\tau)\mathbf{n}(\tau)\cdot\mathbf{J}\right]. \tag{3}$$

Here, $\exp_+$ denotes the exponential with time-ordering, $\Omega(t)$ is the transient angular velocity, $\hat{\mathbf{n}}(t)$ is the unit vector along the rotational direction, and $\mathbf{J} \equiv (\mathsf{J}_x, \mathsf{J}_y, \mathsf{J}_z)$ with $(\mathsf{J}_i)_{jk} = -\epsilon_{ijk}$ is the generator of the SO(3) group. Especially, $\exp(\Omega t \mathsf{J}_z)$ represents the rotation around the z-axis with a constant angular velocity Ω.

Consider the time–dependent unitary transformation

$$|\psi\rangle \longrightarrow |\psi'\rangle = U(t)|\psi\rangle, \tag{4}$$

with

$$U(t) = \exp_-\left[i\int_0^t \mathrm{d}\tau\,\Omega(\tau)\mathbf{n}(\tau)\cdot\hat{\boldsymbol{L}}\right]\exp\left(-im\int_0^t \mathrm{d}\tau\,\frac{\dot{\boldsymbol{\zeta}}_\tau^2}{2}\right) \times \exp\big(-im\dot{\boldsymbol{\zeta}}_t\cdot\hat{\boldsymbol{r}}\big)\exp\big(i\boldsymbol{\zeta}_t\cdot\hat{\boldsymbol{p}}\big). \tag{5}$$

Here, $\hat{\boldsymbol{L}} = \hat{\boldsymbol{r}} \times \hat{\boldsymbol{p}}$ is the angular momentum and $\exp_-$ denotes the exponential with anti-time-ordering, . The Hamiltonian transforms accordingly as

$$H \longrightarrow H' = UHU^\dagger + i\dot{U}U^\dagger. \tag{6}$$

As a result, the new Hamiltonian reads

$$H' = \frac{[\mathsf{R}_t\hat{\boldsymbol{p}} - e\boldsymbol{A}(\hat{\boldsymbol{x}}_t,t)]^2}{2m} + V_0(\hat{\boldsymbol{r}}) + e[\phi(\hat{\boldsymbol{x}}_t,t) - \dot{\boldsymbol{\zeta}}\cdot\boldsymbol{A}(\hat{\boldsymbol{x}}_t,t)] - \Omega(t)\mathbf{n}(t)\cdot\big[(\mathsf{R}_t\hat{\boldsymbol{r}})\times(\mathsf{R}_t\hat{\boldsymbol{p}})\big] + m\ddot{\boldsymbol{\zeta}}_t\cdot(\mathsf{R}_t\hat{\boldsymbol{r}}). \tag{7}$$

where $\hat{\boldsymbol{x}}_t \equiv \mathsf{R}_t\hat{\boldsymbol{r}} + \boldsymbol{\zeta}_t$. See detailed derivations in Appendix A.

B. Electromagnetic field transformation

The dynamics of the electromagnetic (EM) field in the non-inertial frame is governed by the covariant Maxwell's equation,

$$F^{\mu\nu}{}_{;\mu}(x) = 0 \quad \text{and} \quad F_{\mu\nu;\lambda} + F_{\lambda\mu;\nu} + F_{\nu\lambda;\mu} = 0. \tag{8}$$

Here we define the covariant derivative as

$$V^\mu{}_{;\nu} \equiv V^\mu{}_{,\nu} + \Gamma^\mu_{\lambda\nu}V^\lambda \tag{9a}$$

and

$$V_{\mu;\nu} \equiv V_{\mu,\nu} - \Gamma^\lambda_{\mu\nu}V_\lambda \tag{9b}$$

for any 4-vector V^μ and $V_\mu \equiv g_{\mu\nu}V^\nu$ with $g_{\mu\nu}$ being the Riemann metric. In Eq. (9), we defines $f_{,\mu}(x) \equiv \frac{\partial}{\partial x^\mu}f(x)$ for any function $f(x)$ and the Christoffel connection

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\rho}(g_{\mu\rho,\nu} + g_{\nu\rho,\mu} - g_{\mu\nu,\rho}). \tag{10}$$

The Maxwell's equation [Eq. (8)] enables us to introduce the 4-vector potential $A^\mu \equiv (\phi, \boldsymbol{A})$, such that

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} = A_{\nu,\mu} - A_{\mu,\nu}, \tag{11}$$

where the second equality holds due to the symmetry $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$. Consequently, we have

$$A^\nu{}_{;\mu}{}^{;\mu} - A^\mu{}_{;\mu;\nu} + R_{\rho\nu}A^\rho = 0. \tag{12}$$

Besides, the gauge condition is applied together to determine the dynamics of the 4-vector potential.

We consider the transformation as

$$\widetilde{t} = t \quad \text{and} \quad \widetilde{\boldsymbol{r}} = \widetilde{\mathsf{R}}_t^{-1}(\boldsymbol{r} - \widetilde{\boldsymbol{\zeta}}_t) \tag{13}$$

with $g_{\mu\nu}(\boldsymbol{r},t) = g_{\mu\nu}(x) = \eta_{\mu\nu}$. The 4-vector potential transforms as

$$\begin{aligned} &\widetilde{A}^0(\widetilde{x}) = A^0(x), \\ &\widetilde{\boldsymbol{A}}(\widetilde{x}) = \widetilde{\mathsf{R}}_t^{-1}\boldsymbol{A}(x) + \big[\dot{\widetilde{\mathsf{R}}}_t^{-1}(\boldsymbol{r} - \widetilde{\boldsymbol{\zeta}}_t) - \widetilde{\mathsf{R}}_t^{-1}\dot{\widetilde{\boldsymbol{\zeta}}}_t\big]A^0(x). \end{aligned} \tag{14}$$

Special cases include:

1. If the field is comoving with the particle, $\tilde{\mathsf{R}}_t = \mathsf{R}_t$, $\tilde{\boldsymbol{\zeta}}_t = \boldsymbol{\zeta}_t$, leading to $\tilde{\boldsymbol{r}} = \boldsymbol{r}'$ [cf. Eq. (2)];

2. If the field is static, $\tilde{\mathsf{R}}_t = \hat{I}$, $\tilde{\boldsymbol{\zeta}}_t = 0$, leading to $\tilde{\boldsymbol{r}} = \boldsymbol{r}$.

In this work, we quantize the electromagnetic field under the Coulomb gauge, reading

$$\widetilde{\boldsymbol{\nabla}} \cdot \widetilde{\boldsymbol{A}}(\widetilde{\boldsymbol{r}}, t) = 0. \tag{15}$$

This leads to $\widetilde{\phi}(\widetilde{\boldsymbol{r}}, t) = 0$ and

$$\widetilde{\boldsymbol{A}}_{;\mu}{}^{;\mu}(\widetilde{\boldsymbol{r}}, t) = 0. \tag{16}$$

The general solution of Eq. (16) allows us to quantize the vector potential as

$$\widetilde{\boldsymbol{A}}(\tilde{\boldsymbol{r}}, t) = \sum_{k,s} \bar{Z}_k \boldsymbol{\epsilon}_k^s \Big(\hat{a}_k^s \mathrm{e}^{-i\omega_k t + i\boldsymbol{k}\cdot\tilde{\boldsymbol{r}}} + \hat{a}_k^{s\dagger} \mathrm{e}^{i\omega_k t - i\boldsymbol{k}\cdot\tilde{\boldsymbol{r}}} \Big). \tag{17}$$

Here, $\bar{Z}_k$ is the normalization constant, $s = 1, 2$ labels the two polarizing states of photon, and $\boldsymbol{\epsilon}_{\boldsymbol{k}}^s$ are the polarization vectors perpendicular to the wave vector $\boldsymbol{k}$. Note that we can determine $\bar{Z}_k$ in Eq. (16) via the canonical quantization condition. Substituting Eq. (17) into Eq. (16), we can obtain the dispersion relation $\omega_{\boldsymbol{k}}$. One may directly apply the discussions and results of this subsection to other types of environments such as phonons.

III. NON-INERTIAL EFFECTS IN OPEN QUANTUM SYSTEMS

A. Total Hamiltonian with non-inertial motion

Combining the results in Sec. II, we arrive at the total system–plus–bath composite Hamiltonian, reading

$$H' = H_{\mathrm{S}} + H_{\mathrm{SE}} \tag{18}$$

with the system Hamiltonian and the system–environment interaction being

$$H_{\mathrm{S}} = \frac{\hat{\boldsymbol{p}}^2}{2m} + V_0(\hat{\boldsymbol{r}}) - \Omega(t)\mathbf{n}(t) \cdot \mathsf{R}_t \hat{\boldsymbol{L}} + m\ddot{\boldsymbol{\zeta}}_t \cdot (\mathsf{R}_t \hat{\boldsymbol{r}}) \tag{19}$$

and

$$H_{\mathrm{SE}} = -e\tilde{\mathsf{R}}_t^{-1} \left(\frac{\mathsf{R}_t \hat{\boldsymbol{p}}}{m} + \dot{\tilde{\boldsymbol{\zeta}}}_t \right) \cdot \widetilde{\boldsymbol{A}}\big(\tilde{\mathsf{R}}_t^{-1}(\mathsf{R}_t \hat{\boldsymbol{r}} + \boldsymbol{\zeta}_t - \tilde{\boldsymbol{\zeta}}_t)\big), \tag{20}$$

respectively. Here, we express H' in the environment Hamiltonian H_{E}-interaction picture and we ignore the terms with order of e^2. We further adopt the long wavelength approximation for the EM field, i.e., $\boldsymbol{k} \cdot \tilde{\boldsymbol{r}} \ll 1$. This leads to

$$\widetilde{\boldsymbol{A}}(\tilde{\boldsymbol{r}}, t) \simeq \sum_{k,s} \bar{Z}_k \boldsymbol{\epsilon}_k^s \Big(\hat{a}_k^s \mathrm{e}^{-i\omega_k t} + \hat{a}_k^{s\dagger} \mathrm{e}^{i\omega_k t} \Big), \tag{21}$$

which is independent of the coordinate.

B. Spectral density

This subsection gives a comprehensive account on the settings of open quantum system and calculates the environment spectral density. By using the long wavelength approximation, Eq. (20) can be recast into

$$H_{\mathrm{SE}} = -\hat{\boldsymbol{Q}}(t) \cdot \widetilde{\boldsymbol{A}}(t), \tag{22}$$

where we denote $\hat{\boldsymbol{Q}}(t) \equiv e\tilde{\mathsf{R}}_t^{-1}\Big(\frac{\mathsf{R}_t \hat{\boldsymbol{p}}}{m} + \dot{\boldsymbol{\zeta}}\Big)$. The environmental influence on the system is fully characterized by the autocorrelation functions of the vector potential:

$$C_{ij}(t) \equiv \langle \widetilde{A}_i(t) \widetilde{A}_j(0) \rangle_{\mathrm{E}}, \tag{23}$$

where $\widetilde{A}_i$ represents the i-component of $\widetilde{\boldsymbol{A}}$ in a particular spatial coordinate system. For example, $i = r$, θ, z in the cylindrical coordinate system. Here, $\langle(\,\cdot\,)\rangle_{\mathrm{E}} \equiv \mathrm{tr}_{\mathrm{E}}[(\,\cdot\,)\rho_{\mathrm{E}}^0] = \mathrm{tr}_{\mathrm{E}}[(\cdot)e^{-\beta h_{\mathrm{E}}}/Z_{\mathrm{E}}]$ with $Z_{\mathrm{E}} \equiv \mathrm{tr}_{\mathrm{E}}(e^{-\beta h_{\mathrm{E}}})$.

Or equivalently, it can be characterized by the environment spectral density:

$$\begin{aligned} J_{ij}(\omega) &\equiv \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t} \langle [\widetilde{A}_i(t), \widetilde{A}_j(0)] \rangle_{\mathrm{E}} \\ &= \pi \sum_{ks} \bar{Z}_k^2 \epsilon_{ki}^s \epsilon_{kj}^s \Big[\delta(\omega - \omega_k) - \delta(\omega + \omega_k) \Big]. \end{aligned} \tag{24}$$

Here, $\langle(\,\cdot\,)\rangle_{\mathrm{E}} \equiv \mathrm{tr}_{\mathrm{E}}[(\,\cdot\,)\rho_{\mathrm{E}}^0] = \mathrm{tr}_{\mathrm{E}}[(\cdot)e^{-\beta h_{\mathrm{E}}}/Z_{\mathrm{E}}]$ with $Z_{\mathrm{E}} \equiv \mathrm{tr}_{\mathrm{E}}(e^{-\beta h_{\mathrm{E}}})$. They satisfy the positivity relations, $J_{ii}(\omega)/\omega \geq 0$ and $|J_{ij}(\omega)|^2 \leq J_{ii}(\omega) J_{jj}(\omega)$, and the symmetry relations,

$$J_{ij}^*(\omega) = -J_{ij}(-\omega) = J_{ji}(\omega). \tag{25}$$

Together with the detailed balance relation, one can readily obtain [2, 51, 52]

$$\langle \widetilde{A}_i(t) \widetilde{A}_j(0) \rangle_{\mathrm{E}} = \frac{1}{\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega \frac{e^{-i\omega t} J_{ij}(\omega)}{1 - e^{-\beta\omega}}. \tag{26}$$

This is the bosonic fluctuation–dissipation theorem, which relates the bath correlation function to the spectral density function.

According to expression of $H_{\mathrm{SE}}^{(\mathrm{II})}(t)$, the bath can be seen as driven by an external classical field, $\tilde{\mathsf{R}}_t^{-1}\dot{\boldsymbol{\zeta}}$.

C. Dissipaton theory: An exact framework

The HEOM starts with an exponential expansion of Eq. (23),

$$C_{ij}(t) = \sum_{\kappa=1}^{K} \eta_{ij\kappa} e^{-\gamma_\kappa t}, \tag{27}$$

where we set $\gamma_{ij\kappa} = \gamma_\kappa$ for simplicity. This can generally be achieved via certain sum–over–poles expansion on the

Fourier integrand of Eq. (26), followed by the Cauchy's contour integration in the low–half plane.

From the definition of Eq. (23), we obtain the Time-reversal relation,

$$C_{ji}(-t)=[C_{ij}(t)]^{*}=\sum_{\kappa=1}^{K}\eta^{*}_{ij\kappa}e^{-\gamma^{*}_{\kappa}t}=\sum_{\kappa=1}^{K}\eta^{*}_{ij\bar\kappa}e^{-\gamma_{\bar\kappa}t}. \tag{28}$$

The DEOM theory explicitly identifies all involved dynamical variables, $\{\rho^{(n)}_{\mathbf{n}}(t)\}$, as follows. First of all, according to the Gaussian–Wick's thermodynamics theorem [51, 52], the influences of the linearly coupled bath environment are completely characterized by the bare bath correlation functions, Eqs. (27) and (28). Consider now the dissipaton decomposition on the hybridization bath operator [48, 53],

$$\widetilde{A}_i=\sum_{\kappa=1}^{K}\hat f_{i\kappa}. \tag{29}$$

The involving dissipatons that recover Eqs. (27) and (28) are statistically independent quasi–particles, with [48, 53]

$$\begin{aligned}\langle\hat f_{i\kappa}(t)\hat f_{j\kappa'}(0)\rangle_{\rm B}&=\delta_{\kappa\kappa'}\eta_{ij\kappa}e^{-\gamma_\kappa t},\\ \langle\hat f_{j\kappa'}(0)\hat f_{i\kappa}(t)\rangle_{\rm B}&=\delta_{\kappa\kappa'}\eta^{*}_{ij\bar\kappa}e^{-\gamma_\kappa t}.\end{aligned} \tag{30}$$

The above expressions, where $t>0$, highlight the basic feature of dissipatons, whose forward and backward correlations functions in the bare bath ensemble share a common exponent. This feature leads to the *generalized diffusion equation* [48, 53],

$$\mathrm{tr}_{\rm B}\Big[\Big(\frac{\partial}{\partial t}\hat f_{i\kappa}\Big)_{\rm B}\rho_{\rm T}(t)\Big]=-\gamma_\kappa\,\mathrm{tr}_{\rm B}\big[\hat f_{i\kappa}\rho_{\rm T}(t)\big]. \tag{31}$$

While γ_κ can be complex, the total composite $\rho_{\rm T}(t)$ is non-Gaussian in general.

The dynamical variables in DEOM are called the dissipaton density operators (DDOs), defined as [48, 53]:

$$\rho^{(n)}_{\mathbf{n}}(t)\equiv\mathrm{tr}_{\rm B}\Big[\Big(\prod_{i\kappa}\hat f^{n_{i\kappa}}_{i\kappa}\Big)^{\circ}\rho_{\rm T}(t)\Big]. \tag{32}$$

Here, $n=\sum_{i\kappa}n_{i\kappa}$ and $\mathbf{n}=\{n_{i\kappa}\}$ that is an ordered set of the occupation numbers, $n_{i\kappa}=0,1,\cdots$, on individual dissipatons. The circled parentheses, $(\cdots)^{\circ}$, is *irreducible* notation, so that all the c-numbers in the normal ordering of dissipatons product vanish. For bosonic dissipatons it follows that $(\hat f_{i\kappa}\hat f_{j\kappa'})^{\circ}=(\hat f_{j\kappa'}\hat f_{i\kappa})^{\circ}$. In other words, the irreducible product of dissipatons inside $(\cdots)^{\circ}$ in Eq. (32) resembles the second–quantization representation of a bosonic permanent. The DDOs for fermionic coupled environment are similar, but resemble a Slater determinant, having the occupation number of 0 or 1 only, due to the antisymmetric permutation relation [48, 53]. Therefore, $\rho^{(n)}_{\mathbf{n}}(t)$ of Eq. (32) specifies an n–dissipaton configuration, with $\rho^{(0)}_{\mathbf{0}}(t)=\rho_{\rm S}(t)$ being just the reduced system density operator. Denote also $\rho^{(n\pm1)}_{\mathbf{n}^{\pm}_{i\kappa}}$ as the associated $(n\pm1)$-dissipatons configuration, with $\mathbf{n}^{\pm}_{ik}$ differing from $\mathbf{n}$ only at the specified $\hat f_{ik}$-disspaton occupation number, $n_{i\kappa}$, by ±1.

The most important ingredient in the dissipaton algebra is the *generalized Wick's theorem* [48, 53]:

$$\begin{aligned}&\mathrm{tr}_{\rm B}\Big[\Big(\prod_{i\kappa}\hat f^{n_{i\kappa}}_{i\kappa}\Big)^{\circ}\hat f_{j\kappa'}\rho_{\rm T}(t)\Big]\\&=\rho^{(n+1)}_{\mathbf{n}^{+}_{j\kappa'}}(t)+\sum_{i\kappa}n_{i\kappa}\langle\hat f_{i\kappa}\hat f_{j\kappa'}\rangle^{>}_{\rm B}\rho^{(n-1)}_{\mathbf{n}^{-}_{i\kappa}}(t),\end{aligned} \tag{33}$$

and

$$\begin{aligned}&\mathrm{tr}_{\rm B}\Big[\Big(\prod_{i\kappa}\hat f^{n_{i\kappa}}_{i\kappa}\Big)^{\circ}\rho_{\rm T}(t)\hat f_{j\kappa'}\Big]\\&=\rho^{(n+1)}_{\mathbf{n}^{+}_{j\kappa'}}(t)+\sum_{i\kappa}n_{i\kappa}\langle\hat f_{jk'}\hat f_{i\kappa}\rangle^{<}_{\rm B}\rho^{(n-1)}_{\mathbf{n}^{-}_{i\kappa}}(t).\end{aligned} \tag{34}$$

The involved forward $\langle\hat f_{i\kappa}\hat f_{j\kappa'}\rangle^{>}_{\rm B}$ and backward $\langle\hat f_{j\kappa'}\hat f_{i\kappa}\rangle^{<}_{\rm B}$ coefficients are related to the correlation functions in Eq. (30) as

$$\begin{aligned}\langle\hat f_{i\kappa}\hat f_{j\kappa'}\rangle^{>}_{\rm B}&\equiv\langle\hat f_{i\kappa}(0+)\hat f_{j\kappa'}(0)\rangle_{\rm B}=\eta_{ij\kappa}\delta_{\kappa\kappa'},\\ \langle\hat f_{j\kappa'}\hat f_{i\kappa}\rangle^{<}_{\rm B}&\equiv\langle\hat f_{j\kappa'}(0)\hat f_{i\kappa}(0+)\rangle_{\rm B}=\eta^{*}_{ij\bar\kappa}\delta_{\kappa\kappa'}.\end{aligned} \tag{35}$$

The DEOM can now be readily constructed by applying $\dot\rho_{\rm T}(t)=-i[H'(t),\rho_{\rm T}(t)]$, to the total composite density operator in Eq. (32); i.e.,

$$\dot\rho^{(n)}_{\mathbf{n}}(t)=-i\,\mathrm{tr}_{\rm B}\Big\{\Big(\prod_{i\kappa}\hat f^{n_{i\kappa}}_{i\kappa}\Big)^{\circ}[H'(t),\rho_{\rm T}(t)]\Big\}. \tag{36}$$

To proceed, we express the total composite Hamiltonian, Eq. (18) with Eqs. (19), (20), (22) and (29), as

$$H'(t)=H_{\rm S}(t)+\sum_{i\kappa}\hat Q_i(t)\hat f_{i\kappa}. \tag{37}$$

Equation (36) is then evaluated by using Eq. (31) , and Eqs. (33)–(35) for the action of the last term in Eq. (37). We obtain [48, 53]

$$\begin{aligned}\dot\rho^{(n)}_{\mathbf{n}}=&-i[H_{\rm S}(t),\rho^{(n)}_{\mathbf{n}}]-\sum_{i\kappa}n_{i\kappa}\gamma_\kappa\rho^{(n)}_{\mathbf{n}}-i\sum_{i\kappa}[Q_i(t),\rho^{(n+1)}_{\mathbf{n}^{+}_{i\kappa}}]\\&-i\sum_{ij\kappa}n_{i\kappa}\Big[\eta_{ij\kappa}\hat Q_j(t)\rho^{(n-1)}_{\mathbf{n}^{-}_{i\kappa}}-\eta^{*}_{ij\bar\kappa}\rho^{(n-1)}_{\mathbf{n}^{-}_{i\kappa}}\hat Q_j(t)\Big].\end{aligned} \tag{38}$$

This is a temperature–independent real parameter. The contributing coefficients, $\eta_{ij\kappa}$ and $\eta^{*}_{ij\bar\kappa}$, arise solely from the poles of the bath spectral density; see the comments after Eq. (27).

IV. SUMMARY

This work presents a universal formalism of open system quantum mechanics with non-inertial motions. We establish the theory based on a charged system interacting with the electromagnetic field environment. The formulation allows the non-inertial motions of the system and environment to be different. Under the non-inertial unitary transformation, the electromagnetic field still satisfies the Gauss–Wick's statistics, leading to the validity of the dissipaton decomposition of the vector potential $\widetilde{\boldsymbol{A}}$. The corresponding dissipaton equation of motion presents an exact numerical approach to the reduced system dynamics and correlations [48, 53]. Further developments may include extending the formalism to the relativistic Poincaré transformation and applying to exploring the non-inertial effects on open quantum systems, including the cavity QED and chemical reactions.

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Appendix A: Derivation of Eq. (18)

Firstly, we rewrite the unitary transformation in Eq. (5) as

$$U(t) = \exp\left(-im\int_0^t \mathrm{d}\tau\, \frac{\dot{\boldsymbol{\zeta}}_\tau^2}{2}\right)\cdot U_1(t)\cdot U_2(t), \tag{A1}$$

where

$$U_1(t) = \exp_-\left[i\int_0^t \mathrm{d}\tau\, \Omega(\tau)\mathbf{n}(\tau)\cdot\hat{\boldsymbol{L}}\right], \tag{A2a}$$

and

$$U_2(t) = \exp\Big(-im\dot{\boldsymbol{\zeta}}_t\cdot\hat{\boldsymbol{r}}\Big)\cdot\exp\Big(i\boldsymbol{\zeta}_t\cdot\hat{\boldsymbol{p}}\Big). \tag{A2b}$$

Consider the $UHU^\dagger$ term. Evidently,

$$U(t)HU^\dagger(t) = U_1(t)U_2(t)HU_2^\dagger(t)U_1^\dagger(t), \tag{A3}$$

since $\exp\big(-im\int_0^t \mathrm{d}\tau\, \dot{\boldsymbol{\zeta}}_\tau^2/2\big)$ is a pure phase factor. Then by using

$$U_2(t)\hat{\boldsymbol{r}}U_2^\dagger(t) = \hat{\boldsymbol{r}} + \boldsymbol{\zeta}_t, \tag{A4a}$$

$$U_2(t)\hat{\boldsymbol{p}}U_2^\dagger(t) = \hat{\boldsymbol{p}} + m\dot{\boldsymbol{\zeta}}_t, \tag{A4b}$$

we obtain

$$U(t)HU^\dagger(t) = U_1(t)H(\hat{\boldsymbol{r}}+\boldsymbol{\zeta}_t, \hat{\boldsymbol{p}}+m\dot{\boldsymbol{\zeta}}_t)U_1^\dagger(t). \tag{A5}$$

Further noting

$$U_1(t)(\hat{\boldsymbol{r}}+\boldsymbol{\zeta}_t)U_1^\dagger(t) = \mathsf{R}_t\hat{\boldsymbol{r}} + \boldsymbol{\zeta}_t, \tag{A6a}$$

$$U_1(t)(\hat{\boldsymbol{p}}+m\dot{\boldsymbol{\zeta}}_t)U_1^\dagger(t) = \mathsf{R}_t\hat{\boldsymbol{p}} + m\dot{\boldsymbol{\zeta}}_t, \tag{A6b}$$

we have

$$U(t)HU^\dagger(t) = \frac{1}{2m}\big[\mathsf{R}_t\hat{\boldsymbol{p}} + m\dot{\boldsymbol{\zeta}}_t - e\boldsymbol{A}(\mathsf{R}_t\hat{\boldsymbol{r}}+\boldsymbol{\zeta}_t, t)\big]^2 + V_0(\hat{\boldsymbol{r}}) + e\phi(\mathsf{R}_t\hat{\boldsymbol{r}}+\boldsymbol{\zeta}_t, t). \tag{A7}$$

For the $i\dot{U}U^\dagger$ term, according to Eq. (A1), we have

$$i\dot{U}U^\dagger = -m\dot{\boldsymbol{\zeta}}_t^2/2 + m\ddot{\boldsymbol{\zeta}}_t\cdot(\mathsf{R}_t\hat{\boldsymbol{r}}) - \dot{\boldsymbol{\zeta}}_t\cdot(\mathsf{R}_t\hat{\boldsymbol{p}}) - \Omega(t)\mathbf{n}(t)\cdot\big[(\mathsf{R}_t\hat{\boldsymbol{r}})\times(\mathsf{R}_t\hat{\boldsymbol{p}})\big], \tag{A8}$$

where we have used Eqs. (A4) and (A6) with noting that $U_1\hat{\boldsymbol{L}}U_1^\dagger = (\mathsf{R}_t\hat{\boldsymbol{r}})\times(\mathsf{R}_t\hat{\boldsymbol{p}}) = \mathsf{R}_t\hat{\boldsymbol{L}}$. The combination of Eqs. (A5) and (A8) gives rise to the expression of Eq. (18).

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