A try for dark energy in quantum field theory: The vacuum energy of neutrino field

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The quartic-divergent vacuum energy poses an ultraviolet (UV) challenge (the cosmological constant problem) in probing the nature of dark energy. Here we try to evaluate the contribution of the vacuum energy to dark energy with a method of the UV-free scheme. The result indicates that it is not a problem in the UV region but a question of the contributions of heavy fields being suppressed. Then, we explore an effective description via scale decoupling. The parameter spaces suggest that the vacuum energy of active neutrino fields can naturally meet the observation of dark energy density, and a neutrino with a typical mass $\sim$ 10 meV (10^{-3} eV) is expected. The normal ordering neutrinos are preferred by naturalness, and the neutrino mass window set by dark energy is 6.3 meV $\lesssim m_1 \lesssim$ 16.3 meV, 10.7 meV $\lesssim m_2 \lesssim$ 18.4 meV, 50.5 meV $\lesssim m_3 \lesssim$ 52.7 meV.

I. INTRODUCTION

Physics thrives on crisis [1], and the ultraviolet (UV) problem is one of the keys in the development of modern physics. In the context of the UV catastrophe in classical physics, the energy quanta were introduced by Planck [2] while studying blackbody radiation problems, which led to the birth of quantum theory. In quantum field theory (QFT), the problem of UV divergences reappears, that is, loop corrections of a transition process are often UV divergent. It means that the transition amplitude obtained by Feynman rules (the physical input) is not directly equal to the physical result (the physical output). To extract the finite result from a UV-divergent input, the traditional approach is regularization & renormalization, that is, the UV divergence is firstly expressed by the regulator based on equivalent transformation and then removed by counterterms, with the mathematical structure of this route being $\infty - \infty$. This traditional approach is successful in dealing with logarithmic divergences in the standard model (SM). When we leave the logarithmic region and move on, there are UV problems for power-law divergences, i.e.,
(a) The hierarchy problem of the Higgs mass. Loop corrections to the Higgs mass are power-law divergences, with the fine-tuning of the 125 GeV Higgs.
(b) The non-renormalizable Einstein gravity. When one tries to quantize Einstein gravity, an infinite number of counterterms are required for the graviton loops exhibiting power-law divergences.
(c) The cosmological constant problem. The vacuum energy density in QFT is a quartic divergence, and its contribution (Planck scale) exceeds the critical density for a nearly flat universe by 120 orders of magnitude, with the dark energy density $\sim (2.3\ \mathrm{meV})^4$.

How do we deal with the above three UV problems? The traditional approach exhibits limitations in handling power-law divergences, that is, UV divergences under equivalent transformation demand mathematically precarious subtractions via counterterms, especially severe for power-law divergences. In this case, the free flow of ideas is important. Dirac [3] regarded renormalization as mathematically provisional in its handling of UV divergences; the physical laws chosen by nature were found to possess mathematical beauty. Feynman [4] emphasized that mathematical self-consistency and rigor are essential in addressing UV divergences. In the vision of Dyson [5] and Schwinger [6], future approaches to loop corrections would be based on finite quantities. Wilsonian effective theory [7] decouples scales via a physical cutoff that encodes UV effects into low-energy parameters. Hence, is it not time to explore a route that is intrinsically finite? For a new route, it first reproduces the renormalization results for logarithmic divergences, then addresses the three UV problems from power-law divergences.

For loops, assuming the physical contributions being local and insensitive to UV regions, a method of the UV-free scheme [8] is introduced to derive loop contributions, which reproduces the renormalization results for logarithmic divergences. It takes a route of analytic continuation from the physical input $\mathcal{T}_{\mathrm{F}}$ to the physical output $\mathcal{T}_{\mathrm{P}}$ with a mathematical structure $\mathcal{T}_{\mathrm{F}} \to \mathcal{T}_{\mathrm{P}}$,

$$\text{Input}\begin{cases}\textit{Equivalent transformation}\,, \infty-\infty \\ \textit{Analytic continuation}\,,\ \mathcal{T}_{\mathrm{F}} \to \mathcal{T}_{\mathrm{P}}\end{cases}\!\!\Bigg\}\text{Output}\,,$$

and UV divergences are systematically eliminated through $\mathcal{T}_{\mathrm{F}} \to \mathcal{T}_{\mathrm{P}}$ without counterterms. The loop contributions constructed through locally finite quantities are intrinsically free of UV divergences, as guaranteed by locality-based analytic continuation. For power-law divergences, this method inherently obviates UV subtractions and thus interprets the Higgs mass hierarchy problem within SM [8], and an application to Einstein gravity was investigated in Ref. [9]. Here we focus on the UV-divergent vacuum energy. The cosmological constant (dark energy) problem remains a fundamental challenge in physics [1, 10–15]. In exploring the nature of dark energy, a central and unavoidable question is the vacuum energy density ρ_{vac} that corresponds to a cosmological constant ($\Lambda = 8\pi G \rho_{\mathrm{vac}}$). Since the form of the vacuum energy ($\propto \int \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2}$) resembles a loop integral, the UV-free scheme will be employed as a uniform treatment.

II. THE VACUUM ENERGY IN QFT

Dark energy has a negative pressure and plays an important role in the evolution of the universe, whose equation of state is $p_{\mathrm{vac}} = \omega \rho_{\mathrm{vac}}$ with $\omega = -1$ adopted. In QFT, the vacuum

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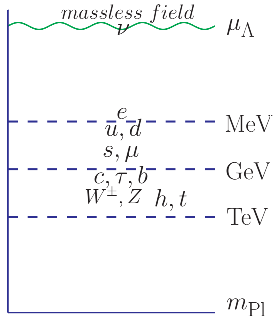

FIG. 1: The vacuum energy in the ripple description.

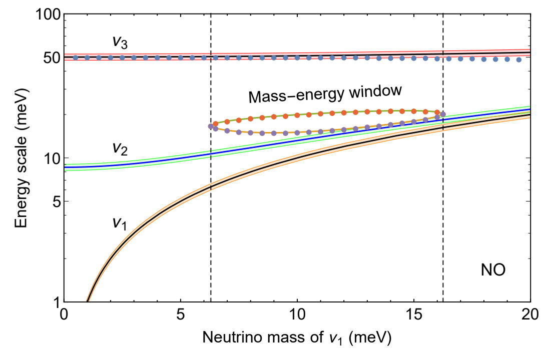

FIG. 2: The normal ordering neutrinos with the dark energy density $(2.3\ \mathrm{meV})^4$. The solid curves are masses of neutrinos with the mass of ν_1 as the input, and the bands with a 5% change in neutrino mass are plotted for a convenient naturalness estimation. The dotted curves are the values of μ_Λ required by the dark energy density with neutrino masses inputted. The region between the two dashed lines is the neutrino mass window.

energy density $\rho_{\rm vac}$ can be obtained by summing the zero-point energy of all fields. For a field i with a mass m_i, its contribution is

$$\rho_0^i = (-1)^{2j_i} g_i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m_i^2}\,, \tag{1}$$

where j_i is the spin of the field, and g_i is the number of degrees of freedom. This is a quartic divergence, yet not a loop contribution. The UV-free scheme [8, 9], a physical analytic continuation of the UV-divergent integral, is adopted to evaluate the contribution of Eq. (1). The Feynman amplitude $\mathcal{T}_{\rm F}^i = \rho_0^i$ is the physical input, and the Feynman-like amplitude $\mathcal{T}_{\rm F}^i(\xi)$ here can be obtained with $\sqrt{k^2+m_i^2}$ replaced by $\sqrt{k^2+m_i^2+\xi}$. The physical transition amplitude $\mathcal{T}_{\rm P}^i$ is

$$\begin{aligned}
\mathcal{T}_{\rm P}^i &= \Big[\int (d\xi)^3 \frac{\partial^3 \mathcal{T}_{\rm F}^i(\xi)}{\partial \xi^3}\Big]_{\xi\to 0} + C \qquad (2)\\
&= \Big[(-1)^{2j_i} g_i \int (d\xi)^3 \int \frac{d^3k}{(2\pi)^3} \frac{3}{16} \frac{1}{\big(\sqrt{k^2+m_i^2+\xi}\big)^5}\Big]_{\xi\to 0} + C \\
&= (-1)^{2j_i} g_i \frac{1}{64\pi^2} m_i^4 \log(m_i^2) + C\ .
\end{aligned}$$

The result with C set by a reference energy scale μ_Λ is

$$\mathcal{T}_{\rm P}^i = (-1)^{2j_i} g_i \frac{1}{64\pi^2} m_i^4 \log \frac{m_i^2}{\mu_\Lambda^2}\,. \tag{3}$$

In the limit $m_i \to 0$, the result is zero for a massless field.[1] The new energy scale μ_Λ is the scale characterizing vacuum energy, analogous to $\Lambda_{\rm QCD}$ for the strong interaction and v for the electroweak sector.

Here is an interpretation of scales in QFT. In a scattering process, the full physical result (tree and loop levels) is independent of the free parameter μ used in perturbative expansions; it can be equivalently described by a tree-level process with an effective coupling at the scale $\mu_{\rm eff}$. The vacuum energy scale μ_Λ is analogous to this physical scale $\mu_{\rm eff}$. Unlike $\mu_{\rm eff}$, which varies with scattering energy, μ_Λ acts as a common reference scale for all field ground states, and may be fixed or slowly running with cosmic evolution.

[1] For another type UV-divergence caused by the discrete sequence, i.e., the Casimir effect from bounded vacuum field modes, the Riemann zeta function $\zeta(-1) = -\frac{1}{12}$ can be adopted as the analytic continuation.

III. THE RIPPLE DESCRIPTION AND NEUTRINO FIELD

The QFT vacuum, the ground state of quantum fields, contains zero-point fluctuations from the uncertainty principle. Their short-distance energy density is given by Eq. (3), showing that a field's contribution to dark energy is not a UV problem. The central question thus becomes how mechanisms beyond current QFT suppress heavy-field contributions. The short-distance result cannot be directly identified with the macroscopic cosmological constant, implying a fundamental scale separation. In the Wilsonian framework [7], this signifies a decoupling between microscopic zero-point energy and its macroscopic effect. We therefore adopt an effective description based on this scale decoupling, taking the total zero-point energy of free field modes as the physical baseline. Physical vacuum energy is then defined as the deviation from this baseline, caused by boundary conditions or interactions. We propose that dark energy originates from the cumulative effect of vacuum fluctuations mediated by carrier fields at the characteristic scale μ_Λ. Here μ_Λ acts as the filtering scale for a field of mass m_i: fields with $m_i \lesssim \mu_\Lambda$ contribute actively, while those with $m_i \gg \mu_\Lambda$ remain inert. This picture is captured by the ripple description (Fig. 1): just as a net water displacement requires coherent ripples, a net physical contribution requires vacuum fluctuations to achieve macroscopic coherence at the scale set by μ_Λ via a fluctuation factor structuring the carrier field, which selects coherent contributions on macroscopic scales, analogous to how the Boltzmann factor selects energy quanta in thermal radiation. Given the Hubble parameter $H_0 \approx 70$ km s^{-1} Mpc^{-1} and dark energy at 71% of the critical density $3H_0^2/8\pi G$, the dark energy density is about $(2.3\ \mathrm{meV})^4$, corresponding to $\mu_\Lambda \sim 10-100$ meV, indicating sensitivity to the neutrino field, in particular to neutrino mass.[2] Considering Eq. (3) and dark energy density, a typical neutrino mass is $m_\nu^{\rm tp} \sim 2.3 \times (32\pi^2)^{\frac{1}{4}} \approx$ 9.7 meV (here $|\log \frac{m_i^2}{\mu_\Lambda^2}|^{\frac{1}{4}} \sim 1$). As a rough prediction, there should be a neutrino with a mass $\sim$ 10 meV.

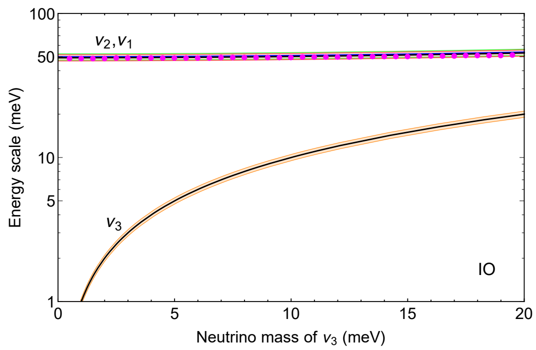

FIG. 3: The inverted ordering neutrinos with the dark energy density $(2.3\ \mathrm{meV})^4$. The solid curves are masses of neutrinos with the mass of ν_3 as the input. The dotted curve is the values of μ_Λ required by the dark energy density.

Now we turn to neutrino masses. The masses of three neutrinos are m_1, m_2 and m_3, and the mass differences between them can be extracted by neutrino oscillations. The mass-splitting results [22] are $\Delta m_{21}^2 \approx 7.41 \times 10^{-5}\mathrm{eV}^2$, $\Delta m_{31}^2 \sim \Delta m_{32}^2 \approx 2.437 \times 10^{-3}\mathrm{eV}^2$ (normal ordering) and $\Delta m_{31}^2 \sim \Delta m_{32}^2 \approx -2.498\times 10^{-3}\mathrm{eV}^2$ (inverted ordering). Considering the naturalness with an effective mass $m_\nu^{\rm tp}$, the quasidegenerate spectrum with $m_1 \simeq m_2 \simeq m_3 \gg \sqrt{|\Delta m_{32}^2|}$ is not favored. It is possible to give an estimate on the mass spectrum of neutrinos. Here we consider a specific implementation of the ripple description by a test fluctuation factor $e^{-m_i^2/\mu_\Lambda^2}$ in Gaussian distribution (see the Appendix) to characterize the active contribution of a field, and g_i together with this factor can be taken as an effective active degrees of freedom g_i^* with $g_i^* = g_i e^{-m_i^2/\mu_\Lambda^2}$ ($\rho_{\rm vac} = \sum_i (-1)^{2j_i} g_i^* \frac{m_i^4}{64\pi^2} \log \frac{m_i^2}{\mu_\Lambda^2}$). For a field with a mass $m_i \gg \mu_\Lambda$, its contribution to the dark energy density is negligible due to the tiny value of g_i^* (this is similar to the freezing of effective degrees of freedom of heavy particles at a low temperature in the early universe). The hierarchical spectra of neutrinos for normal ordering and inverted ordering with the required dark energy density are shown in Fig. 2 and Fig. 3, respectively. In Fig. 3, considering the naturalness of μ_Λ, the inverted ordering is not favored. For the normal ordering shown in Fig. 2, there is a region that neutrino masses at given μ_Λ can naturally produce the physical vacuum energy required by the dark energy density, i.e., the parameter spaces forming a mass-energy window. Likewise, the neutrino mass window set by dark energy is 6.3 meV $\lesssim m_1 \lesssim$ 16.3 meV, 10.7 meV $\lesssim m_2 \lesssim$ 18.4 meV, 50.5 meV $\lesssim m_3 \lesssim$ 52.7 meV, and the total neutrino mass is 67.5 meV $\lesssim m_1 + m_2 + m_3 \lesssim$ 87.4 meV. This mass window can be tested by the future experiments.

Let us give a brief discussion about the Hubble tension. A slightly higher $H_0 = (73.04 \pm 1.04)$ km s^{-1} Mpc^{-1} [23, 24] from the nearby universe and a slightly lower $H_0 = (67.4 \pm 0.5)$ km s^{-1} Mpc^{-1} [25] from the early universe indicate a possible Hubble tension (the tension may be alleviated [26]). For $H_0 \approx 67.4$ km s^{-1} Mpc^{-1} and $H_0 \approx 73$ km s^{-1} Mpc^{-1}, the corresponding dark energy densities are about $(2.24\ \mathrm{meV})^4$ and $(2.37\ \mathrm{meV})^4$, respectively. If the scale μ_Λ has a slow-running behavior and traverses the upper part of the mass-energy window in Fig. 2, the Hubble tension can be explained by this slow-running behavior, i.e., a running cosmological constant. If μ_Λ is fixed, new explanations and mechanisms need to be taken into account.

IV. CONCLUSION AND DISCUSSION

In this paper, the power-law divergence of vacuum energy is established. The physical result obtained in the UV-free scheme indicates it is not a problem in the UV region. Freed from the veil of UV divergence, the core question becomes how contributions of heavy fields are suppressed by unknown mechanisms, constituting a new open question. This work thereby recasts the cosmological constant problem from a fine-tuning puzzle into a defined physical question, thus clarifying the actual nature of the problem. In fundamental physics, such a redefinition is itself an advance, while its full resolution is the next open goal. A new framework comes with its necessary transformation of perspective. Accordingly, we address two recurrent questions (also noted in Ref. [27]): why the value of μ_Λ is chosen at the meV scale, and why contributions from heavy fields are neglected. To the first question about the value of μ_Λ, it is currently lacking pathways to derive its specific value from fundamental principles (e.g., the values of particle masses and coupling constants in SM). Here μ_Λ is taken as an input parameter, and its parameter space is derived by the observation of dark energy density with naturalness. Theoretically, this scale may link to the inflationary field or neutrino mass origin, etc. A connection to neutrinos is of particular interest, as their mass scale would interface quantum vacuum physics with the cosmos. To the second question, this work explores the $\mu_\Lambda-$characterized scale decoupling of vacuum energy from microscopic quantity to macroscopic effect, implying heavy fields remain inert. This suggests that the dark energy problem could be minimally addressed by introducing the new scale μ_Λ into SM.

To account for the dark energy density, a neutrino with a typical mass around 10 meV is expected in the ripple description. Considering the naturalness at the scale μ_Λ, the normal hierarchical spectrum of neutrinos is preferred, and there is a mass-energy window for the neutrino mass spectrum and the dark energy density. The neutrino mass window set by dark energy can be examined by the future experiments. Moreover, for UV-divergence inputs in QFT, there may be another route

[2] Other phenomenological proposals explore neutrino contributions via, e.g., the relic neutrinos with varying masses [16–18], the slow-roll quintessence with extended neutrino sector [19, 20], the cosmological axion and neutrino [21], etc.

besides the routes mentioned in this article. All roads lead to the same goal. With the joint explorations of multiple routes, we hope it will be possible to get close to the harmonious and unified physical structure hidden beneath UV divergences. We look forward to more explorations.

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Appendix: Fluctuation factor

For a mass m carrier field, the $\mu_\Lambda-$characterized fluctuation factor describes macroscopic effects from cumulative vacuum fluctuations. Its potential form is derived as follows. The first is basic assumptions, i.e.,
(a) Independence assumption. Consider a system with N independent random fluctuations $\{q_i\}_{i=1}^N$, satisfying: Zero expectation, with $E[q_i] = 0$; Finite variance, with $Var(q_i) = \sigma_i^2$ (bounded fluctuation strength); Statistical independence between random variables q_i.
(b) Linear superposition. The total cumulative effect X is a linear superposition of individual fluctuations with $X = \sum_{i=1}^N a_i q_i$, where a_i being weight coefficients, and the system exhibits no significant nonlinear coupling.
Next is the derivation via the characteristic function. For each q_i, its characteristic function $\phi_i(\tau) = E[e^{i\tau q_i}]$ expands to $\phi_i(\tau) = 1 + i\tau E[q_i] - \frac{\tau^2}{2}E[q_i^2] + \cdots \approx 1 - \frac{\sigma_i^2\tau^2}{2}$, where $\cdots$ denotes higher-order terms. By independence, the characteristic function of X is the product of individual characteristic functions, $\phi_X(\tau) = \prod_{i=1}^N \phi_i(a_i\tau) \approx \prod_{i=1}^N(1 - \frac{a_i^2\sigma_i^2\tau^2}{2})$. Taking the logarithm and linearizing (using $\ln(1+x) \approx x$ for $x \to 0$), the result is $\ln\phi_X(\tau) \approx -\frac{\tau^2}{2}\sum_{i=1}^N a_i^2\sigma_i^2 \Rightarrow \phi_X(\tau) = \exp(-\frac{\sigma_X^2\tau^2}{2})$, where $\sigma_X^2 = \sum_{i=1}^N a_i^2\sigma_i^2$. This is the characteristic function of a Gaussian distribution. In this paper, we assume that the cumulative effect X of vacuum fluctuations manifests macroscopic effects through carrier fields. Specifically, the fluctuation factor $f(m)$ is set by $\phi_X(\tau)$, and it can be obtained via inverse Fourier transform of $\phi_X(\tau)$ with a matching condition $X \to m$, that is, $f(m) = A\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\tau m}\phi_X(\tau)\,d\tau = \frac{A}{\sqrt{2\pi\sigma_X^2}}\exp(-\frac{m^2}{2\sigma_X^2}) = B\exp(-\frac{m^2\pi B^2}{A^2})$, with $B = \frac{A}{\sqrt{2\pi\sigma_X^2}}$. Considering photon field as the carrier field for normalization, one has $f(0) = 1$, and thus $B = 1$ is obtained. Taking $A^2/\pi = 2\sigma_X^2 = \frac{1}{k}\mu_\Lambda^2$, the fluctuation factor is written as $f(m) = e^{-km^2/\mu_\Lambda^2}$, and $k \sim 1$ can be adopted for $\mu_\Lambda-$characterized fluctuations.

From another perspective, the macroscopic effect of physical vacuum energy requires a long-range description with characteristic scale $\sim 1/\mu_\Lambda$. This reflects the filtering of microscopic energy into a macroscopic effective quantity, a principle common to statistical physics. A classic example is gas pressure, which stems not from each molecule's rest energy mc^2, but from the small fraction in statistically averaged translational kinetic energy. Thus, macroscopic observables are only scale-selected subsets of microscopic energies.

In this analogy, the dark energy density is a macroscopic observable governed by a μ_Λ-scale filter. The microscopic zero-point energy of a quantum field is enormous, yet only fluctuation modes coherent over $\sim 1/\mu_\Lambda$ contribute effectively. For a field with $m \gg \mu_\Lambda$, its coherence length (the Compton wavelength $\lambda_c \sim 1/m$) is far smaller than $1/\mu_\Lambda$. Consequently, a macroscopic volume of size $\sim 1/\mu_\Lambda$ contains many independent fluctuation patches with random phases; their contributions cancel under macroscopic averaging via a random-walk process, leading to exponential suppression. Thus, the cumulative effect of a heavy field is strongly suppressed. The observed small dark energy density therefore does not contradict huge microscopic energies, but shows the cosmos is only sensitive to modes passing the decoupling filter $\lambda_c \gtrsim 1/\mu_\Lambda$. This phase-statistical filtering provides a statistical explanation for the smallness of dark energy density.

[1] S. Weinberg, Rev. Mod. Phys. **61**, 1-23 (1989)
[2] M. Planck, Annalen Phys. **4**, 553 (1901)
[3] P. A. M. Dirac, The Evolution of the Physicist's Picture of Nature, Scientific American **208**, 45-53 (1963); P. A. M. Dirac, Directions in Physics, John Wiley & Sons, New York, 1978; P. A. M. Dirac, The Relation between Mathematics and Physics, 1939, Published in: Proceedings of the Royal Society of Edinburgh **59**, 122-129 (1940).
[4] R. P. Feynman, QED: The Strange Theory of Light and Matter, Princeton University Press, Princeton, NJ, 1985, 1988.
[5] F. J. Dyson, Phys. Rev. **75**, 1736-1755 (1949).
[6] J. Schwinger, Renormalization theory of quantum electrodynamics: An individual view, 1983.
[7] K. G. Wilson, Phys. Rev. B **4**, 3174-3183 (1971); K. G. Wilson, Phys. Rev. B **4**, 3184-3205 (1971); K. G. Wilson and J. B. Kogut, Phys. Rept. **12**, 75-199 (1974).
[8] L. B. Jia, Commun. Theor. Phys. **78** (2026) no.1, 015201 [arXiv:2305.18104 [hep-ph]].
[9] L. B. Jia, [arXiv:2403.09487 [hep-ph]].
[10] A. G. Riess *et al.* [Supernova Search Team], Astron. J. **116**, 1009-1038 (1998) [arXiv:astro-ph/9805201 [astro-ph]].
[11] S. Perlmutter *et al.* [Supernova Cosmology Project], Astrophys. J. **517**, 565-586 (1999) [arXiv:astro-ph/9812133 [astro-ph]].
[12] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559-606 (2003) [arXiv:astro-ph/0207347 [astro-ph]].
[13] T. Padmanabhan, Phys. Rept. **380**, 235-320 (2003) [arXiv:hep-th/0212290 [hep-th]].
[14] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753-1936 (2006) [arXiv:hep-th/0603057 [hep-th]].
[15] M. Li, X. D. Li, S. Wang and Y. Wang, Commun. Theor. Phys. **56** (2011), 525-604 [arXiv:1103.5870 [astro-ph.CO]].
[16] R. Fardon, A. E. Nelson and N. Weiner, JCAP **10**, 005 (2004)

[arXiv:astro-ph/0309800 [astro-ph]].
[17] R. Fardon, A. E. Nelson and N. Weiner, JHEP **03**, 042 (2006) [arXiv:hep-ph/0507235 [hep-ph]].
[18] R. Takahashi and M. Tanimoto, Phys. Lett. B **633**, 675-680 (2006) [arXiv:hep-ph/0507142 [hep-ph]].
[19] P. Q. Hung, [arXiv:hep-ph/0010126 [hep-ph]].
[20] R. Barbieri, L. J. Hall, S. J. Oliver and A. Strumia, Phys. Lett. B **625**, 189-195 (2005) [arXiv:hep-ph/0505124 [hep-ph]].
[21] H. J. de Vega and N. G. Sanchez, Universe **9**, no.4, 167 (2023).
[22] S. Navas *et al.* [Particle Data Group], Phys. Rev. D **110**, no.3, 030001 (2024).
[23] A. G. Riess, L. M. Macri, S. L. Hoffmann, D. Scolnic, S. Casertano, A. V. Filippenko, B. E. Tucker, M. J. Reid, D. O. Jones and J. M. Silverman, *et al.* Astrophys. J. **826**, no.1, 56 (2016) [arXiv:1604.01424 [astro-ph.CO]].
[24] A. G. Riess, W. Yuan, L. M. Macri, D. Scolnic, D. Brout, S. Casertano, D. O. Jones, Y. Murakami, L. Breuval and T. G. Brink, *et al.* Astrophys. J. Lett. **934**, no.1, L7 (2022) [arXiv:2112.04510 [astro-ph.CO]].
[25] N. Aghanim *et al.* [Planck], Astron. Astrophys. **641**, A6 (2020) [erratum: Astron. Astrophys. **652**, C4 (2021)] [arXiv:1807.06209 [astro-ph.CO]].
[26] W. L. Freedman, B. F. Madore, I. S. Jang, T. J. Hoyt, A. J. Lee and K. A. Owens, [arXiv:2408.06153 [astro-ph.CO]].
[27] J. M. Cline, [arXiv:2410.10522 [hep-ph]].