

Supplemental Material for “Catalog of Altermagnetism in Magnetic Wallpaper/Space Groups and Nonsymmorphic Altermagnets”

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I. TIGHT-BINDING MODEL FOR P1M'1 AND AWG P4'GM'

In this section, we construct explicit tight-binding models for two altermagnetic wallpaper groups (AWGs), P1m'1 and P4'gm', to illustrate the symmetry mechanisms of the main text. The first realizes a conventional d -wave altermagnet, while the second is the only nonsymmorphic altermagnet in two dimensions, in which a glide mirror enforces an hourglass-shaped spin splitting.

A. AWG P1m'1

We consider a two-dimensional altermagnetic crystal lattice characterized by the altermagnetic wallpaper group P1m'1, as shown in Fig.3(a) of main text. We take the lattice vectors $\mathbf{a}_1 = (4, 0)$ and $\mathbf{a}_2 = (0, 6)$, and the magnetic and nonmagnetic atoms occupy the positions $\vec{r}_1 = (x, y)$, $\vec{r}_2 = (-x, y)$ and $\vec{r}_3 = (0, 0)$, where x and y are fractional coordinates. The spin-flipping mirror M'_x links the opposite-spin sites. Together with the spinless time-reversal symmetry S , it yields the effective spin-flipping mirror $\widetilde{M}'_y = SM'_x$. These two mirror symmetries protect the spin-up and spin-down degeneracy along the x and y axes. The corresponding effective model for the crystal lattice then takes the form

$$\mathcal{H}(k_x, k_y) = \begin{pmatrix} M & h_{12}(k_x, k_y) & h_{13}(k_x, k_y) & 0 & 0 & 0 \\ h_{12}^*(k_x, k_y) & -M & h_{23}(k_x, k_y) & 0 & 0 & 0 \\ h_{13}^*(k_x, k_y) & h_{23}^*(k_x, k_y) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -M & h_{12}(k_x, k_y) & h_{13}(k_x, k_y) \\ 0 & 0 & 0 & h_{12}^*(k_x, k_y) & M & h_{23}(k_x, k_y) \\ 0 & 0 & 0 & h_{13}^*(k_x, k_y) & h_{23}^*(k_x, k_y) & 0 \end{pmatrix}, \quad (1)$$

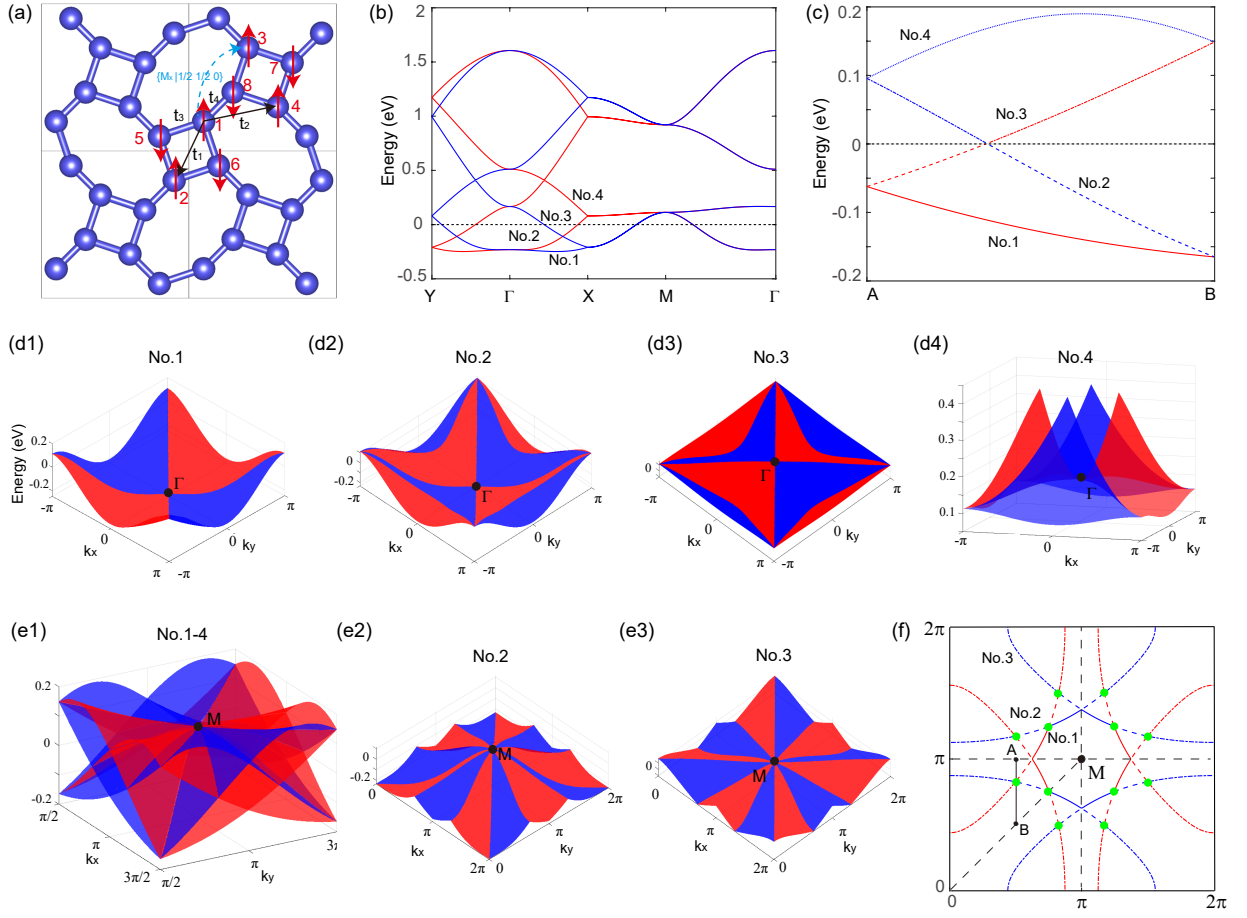


FIG. S1: (color online) (a) Two-dimensional crystal lattice of AWG $P4'gm'$, where red arrows denote spin-up and spin-down moments. (b) Band structure along high-symmetry lines; red and blue lines denote spin-up and spin-down bands, with the four bands nearest the Fermi level labeled No. 1–4. (c) Hourglass-shaped bands formed by bands No. 1–4 along the momentum path AB, marked in (f). (d1)–(d4) Three-dimensional energy dispersions of bands No. 1–4 over the Brillouin zone, centered at the Γ point. (e1) The four intertwined bands near the M point; (e2) and (e3) the spin textures of the inner bands No. 2 and No. 3 near M . (f) Calculated Fermi surface, with green dots marking spin-degenerate crossing points. Fractional coordinates are chosen as $x = 0.1$ and $y = 0.2$. The model parameters are $t_1 = 0.261t_0$, $t_2 = 0.217t_0$, $t_3 = 0.387t_0$, $t_4 = 0.428t_0$, $M = 0.8t_0$, $\mu = 0.533t_0$ in units of t_0 .

where the basis operator is denoted by $\psi_{\mathbf{k}\sigma}^\dagger = [c_{1\uparrow}^\dagger(\mathbf{k}), c_{2\uparrow}^\dagger(\mathbf{k}), c_{3\uparrow}^\dagger(\mathbf{k}), c_{1\downarrow}^\dagger(\mathbf{k}), c_{2\downarrow}^\dagger(\mathbf{k}), c_{3\downarrow}^\dagger(\mathbf{k})]$, and M denotes the magnetization. The matrix elements $h(\mathbf{k})$ are given by

$$\begin{aligned} h_{12}(k_x, k_y) &= t_1 e^{i2xk_x} + t_2 e^{i(2x-1)k_x}, \quad h_{13}(k_x, k_y) = t_3 e^{i(xk_x + yk_y)} + t_4 e^{i(xk_x + (y-1)k_y)} + t_5 e^{i((x-1)k_x + yk_y)}, \\ h_{23}(k_x, k_y) &= h_{13}(-k_x, k_y), \end{aligned} \quad (2)$$

Here, the hopping parameter is taken as $t_i = t_0 e^{-\delta_{r_i}}$, where δ_{r_i} is the distance between the two corresponding atoms in Fig.3(a) of main text. We fix $t_0 = 1$ and use it as the unit of energy.

B. AWG $P4'gm'$

We next consider a two-dimensional altermagnetic crystal lattice characterized by the altermagnetic wallpaper group $P4'gm'$ (the only nonsymmorphic altermagnet in two dimensions), as shown in Fig. S1(a). The lattice vectors are $\mathbf{a}_1 = (3, 0)$ and $\mathbf{a}_2 = (0, 3)$, and the magnetic atoms occupy the positions

$$\begin{aligned} \vec{r}_1 &= (x, y), \quad \vec{r}_2 = (-x, -y), \quad \vec{r}_3 = (-x + 1/2, y + 1/2), \quad \vec{r}_4 = (x + 1/2, -y + 1/2), \\ \vec{r}_5 &= (-y, x), \quad \vec{r}_6 = (y, -x), \quad \vec{r}_7 = (y + 1/2, x + 1/2), \quad \vec{r}_8 = (-y + 1/2, -x + 1/2) \end{aligned} \quad (3)$$

where x and y are fractional coordinates; atoms No. 1–4 carry spin-up moments while No. 5–8 host spin-down moments. The spin-conserving glide mirrors $\{M_{x/y}|1/2\ 1/2\ 0\}$ link the same-spin sites and, together with the spinless time-reversal symmetry S , enforce the same-spin degeneracy along the BZ boundary. In contrast, the diagonal mirrors $\{M'_{xy/x\bar{y}}|1/2\ 1/2\ 0\}$ exchange sublattices with opposite spin, thereby protecting the spin-up and spin-down degeneracy along the BZ diagonals. The corresponding effective model then takes the form

$$H = \sum_{l,l',\mathbf{k},\sigma} c_{l,\mathbf{k},\sigma}^\dagger (\mathcal{H}_{\mathbf{k},\sigma})_{l,l'} c_{l',\mathbf{k},\sigma} + M \sum_{l,\mathbf{k},\sigma} \alpha_l c_{l,\mathbf{k},\sigma}^\dagger c_{l,\mathbf{k},\sigma} \sigma_z - \mu \sum_{l,\mathbf{k},\sigma} c_{l,\mathbf{k},\sigma}^\dagger c_{l,\mathbf{k},\sigma}, \quad (4)$$

where $c_{l,\mathbf{k},\sigma}^\dagger$ creates a fermion with spin σ on sublattice l of the eight-site magnetic unit cell. The chemical potential μ controls the filling, M denotes the magnetization, and $\alpha_l = 1$ for $l = 1$ –4 while $\alpha_l = -1$ for $l = 5$ –8. The nonmagnetic term $\mathcal{H}_{\mathbf{k},\sigma}$ is

$$\mathcal{H}_{\mathbf{k},\sigma} = \begin{pmatrix} 0 & h_{12}(\mathbf{k}) & 0 & h_{14}(\mathbf{k}) & h_{15}(\mathbf{k}) & h_{16}(\mathbf{k}) & 0 & h_{18}(\mathbf{k}) \\ h_{12}^*(\mathbf{k}) & 0 & h_{23}(\mathbf{k}) & 0 & h_{25}(\mathbf{k}) & h_{26}(\mathbf{k}) & h_{27}(\mathbf{k}) & 0 \\ 0 & h_{23}^*(\mathbf{k}) & 0 & h_{34}(\mathbf{k}) & 0 & h_{36}(\mathbf{k}) & h_{37}(\mathbf{k}) & h_{38}(\mathbf{k}) \\ h_{14}^*(\mathbf{k}) & 0 & h_{34}^*(\mathbf{k}) & 0 & h_{45}(\mathbf{k}) & 0 & h_{47}(\mathbf{k}) & h_{48}(\mathbf{k}) \\ h_{15}^*(\mathbf{k}) & h_{25}^*(\mathbf{k}) & 0 & h_{45}^*(\mathbf{k}) & 0 & h_{56}(\mathbf{k}) & h_{57}(\mathbf{k}) & 0 \\ h_{16}^*(\mathbf{k}) & h_{26}^*(\mathbf{k}) & h_{36}^*(\mathbf{k}) & 0 & h_{56}^*(\mathbf{k}) & 0 & 0 & h_{68}(\mathbf{k}) \\ 0 & h_{27}^*(\mathbf{k}) & h_{37}^*(\mathbf{k}) & h_{47}^*(\mathbf{k}) & h_{57}^*(\mathbf{k}) & 0 & 0 & h_{78}(\mathbf{k}) \\ h_{18}^*(\mathbf{k}) & 0 & h_{38}^*(\mathbf{k}) & h_{48}^*(\mathbf{k}) & 0 & h_{68}^*(\mathbf{k}) & h_{78}^*(\mathbf{k}) & 0 \end{pmatrix}, \quad (5)$$

where the matrix elements are given by

$$\begin{aligned} h_{12}(\mathbf{k}) &= t_1 e^{2i(xk_x + yk_y)}, \quad h_{14}(\mathbf{k}) = 2t_2 \cos \frac{k_x}{2} e^{i(2y - \frac{1}{2})k_y}, \quad h_{15}(\mathbf{k}) = t_3 e^{i[(x+y)k_x + (y-x)k_y]}, \\ h_{16}(\mathbf{k}) &= t_3 e^{i[(x-y)k_x + (y+x)k_y]}, \quad h_{18}(\mathbf{k}) = t_4 e^{i[(x+y - \frac{1}{2})k_x + (y+x - \frac{1}{2})k_y]}, \\ h_{23}(\mathbf{k}) &= 2t_2 \cos \frac{k_x}{2} e^{i(-2y + \frac{1}{2})k_y}, \quad h_{25}(\mathbf{k}) = t_3 e^{i[(-x+y)k_x + (-y-x)k_y]}, \quad h_{26}(\mathbf{k}) = t_3 e^{i[(-x-y)k_x + (-y+x)k_y]}, \\ h_{27}(\mathbf{k}) &= t_4 e^{i[(-x-y + \frac{1}{2})k_x + (-y-x + \frac{1}{2})k_y]}, \quad h_{34}(\mathbf{k}) = t_1 e^{i[-2xk_x + 2yk_y]}, \\ h_{36}(\mathbf{k}) &= t_4 e^{i[(-x-y + \frac{1}{2})k_x + (y-x + \frac{1}{2})k_y]}, \quad h_{37}(\mathbf{k}) = t_3 e^{i[(-x+y)k_x + (y-x)k_y]}, \quad h_{38}(\mathbf{k}) = t_3 e^{i[(-x+y)k_x + (y+x)k_y]}, \\ h_{45}(\mathbf{k}) &= t_4 e^{i[(x+y - \frac{1}{2})k_x - (y+x - \frac{1}{2})k_y]}, \quad h_{47}(\mathbf{k}) = t_3 e^{i[(x-y)k_x + (-y-x)k_y]}, \quad h_{48}(\mathbf{k}) = t_3 e^{i[(x+y)k_x + (-y+x)k_y]}, \\ h_{56}(\mathbf{k}) &= t_1 e^{-2i(yk_x - xk_y)}, \quad h_{57}(\mathbf{k}) = 2t_2 \cos \frac{k_y}{2} e^{i(-2y + \frac{1}{2})k_x}, \quad h_{68}(\mathbf{k}) = 2t_2 \cos \frac{k_y}{2} e^{i(2y - \frac{1}{2})k_x}, \quad h_{78}(\mathbf{k}) = t_1 e^{2i(yk_x + xk_y)} \end{aligned} \quad (6)$$

Here, the hopping parameter is taken as $t_i = t_0 e^{-\delta_{r_i}}$, where δ_{r_i} is the distance between the two corresponding atoms in Fig. S1(a). We fix $t_0 = 1$ and use it as the unit of energy. This is the eight-band tight-binding model referenced in Sec. 4.1 of the main text; we use it to confirm that the effective Hamiltonians H_Γ and H_M derived there capture the nonsymmorphic physics of P4'gm'. Of the sixteen bands (eight per spin sector), the four nearest the Fermi level (labeled No. 1–4 in Fig. S1) realize the four-band effective model, with the outer pair (No. 1, 4) playing the role of the 1st and 4th bands and the inner pair (No. 2, 3) the 2nd and 3rd.

Fig. S1(b) displays the band structure. Consistent with the symmetry analysis, the same-spin degeneracy appears along the BZ boundary, while the opposite-spin degeneracy appears along the diagonals, so the connecting path crosses an hourglass-shaped dispersion [Fig. S1(c)]. Throughout, the outer bands (No. 1, 4) keep a simple d -wave splitting, whereas the inner bands (No. 2, 3) develop a more complex spin texture [Fig. S1(d1)–(d4)]. Near Γ , where the hourglass is absent, the four bands separate into two non-crossing d -wave pairs, reproducing the two decoupled d -wave blocks of H_Γ [Eq. (14) of the main text] in the limit $f \ll g$. Near M, in contrast, they are forced to intertwine [Fig. S1(e1)], realizing H_M [Eq. (15) of the main text]: two identical d -wave two-band models rotated by $2\theta_0$ and pinned to the same energy by the glide mirror M_x , which forbids their separation. Encircling M, the inner pair inverts its spin character 12 times [Fig. S1(e2) and (e3)], producing 12 spin-degenerate nodal lines—four along the diagonals from No. 1–2 crossings and eight from No. 2–3 crossings.

When the Fermi level cuts these intertwined bands, the Fermi surface [Fig. S1(f)] exhibits 12 spin-crossing points. Although this superficially resembles an i -wave altermagnet, its origin is fundamentally different: it arises from two iso-energetic d -wave splittings—the $2\theta_0$ -rotated blocks of H_M —coexisting at the same energy, rather than a single band pair carrying i -wave ($\ell = 6$) harmonics. The inner-ring crossings stem from the No. 1–2 bands and the outer-ring crossings from the No. 2–3 bands, so the full Fermi surface reflects the interference of two d -wave splittings.

II. SPIN DISTRIBUTIONS AND UNLOCKED NODAL PLANES

In this section, we provide additional details on the Brillouin-zone (BZ) spin textures and nodal structures discussed in the main text. The discussion is organized as follows. First, we show that the BZ geometry is not uniquely fixed by the Bravais-lattice type, but also depends on the lattice parameters; consequently, a single Bravais lattice can host distinct BZ configurations. We then illustrate how the corresponding altermagnetic spin distributions appear in these different BZ geometries. Second, we address the non-uniqueness of the principal axis in orthorhombic systems. Different choices of the halving subgroup select different principal axes and therefore yield distinct altermagnetic BZs; these are equivalent orientational representations of the same altermagnetic type, differing only by the orientation of the spin texture. Third, we analyze unlocked nodal planes in monoclinic systems. In the absence of vertical mirror symmetries, spin-degenerate nodal planes are no longer pinned to fixed high-symmetry planes. Nevertheless, the remaining C_{2z} symmetry protects spin-degenerate momentum lines, which constrain the allowed configurations of the unlocked nodal planes.

A. Spin Distributions in Different Brillouin-Zone Geometries

In the main text, we present one representative Brillouin-zone (BZ) spin-distribution pattern for each Bravais lattice, in order to illustrate the spin-splitting features of the corresponding altermagnetic type. However, the BZ geometry is not uniquely determined by the Bravais-lattice type alone, but further depends on the relative lattice parameters and angular constraints¹⁹. Table S1 summarizes the distinct BZ configurations associated with the 14 Bravais lattices, together with their standard labels¹⁹. Among the 14 Bravais lattices, only five admit more than one BZ geometry: the simple triclinic (TRI), C-centered monoclinic (MCLC), face-centered orthorhombic (ORCF), body-centered tetragonal (BCT), and rhombohedral (RHL) lattices; every other Bravais lattice possesses a unique BZ shape. Although TRI admits four BZ geometries, it does not support altermagnetic spin splitting (Table S1). For example, the MCLC lattice can be further divided into five different BZ shapes, MCLC₁–MCLC₅, depending on the relative ratios of the lattice parameters (a, b, c) and the monoclinic angle β . For completeness, Fig. S2 displays the spin-splitting textures in additional distinct BZ geometries, complementing the representative BZs shown in the main text. In these figures, the z direction is uniformly chosen as the principal axis to define the corresponding altermagnetic type.

B. Spin Distributions for Different Choices of the Principal Axis in Orthorhombic Systems

In the main text, we use the point group C_{2v} as an example to illustrate that, in orthorhombic systems, the principal axis used to define an altermagnetic type is not unique. For clarity, the classification table in the main text includes only representative spin-distribution patterns with the z and x axes chosen as the principal axes.

This non-uniqueness originates from the freedom in choosing the halving subgroup: different halving subgroups select different principal axes, which in turn rotate the spin texture in the BZ. For example, for the point group D_{2h} , choosing the x , y , or z axis as the principal axis gives the halving subgroups $C_{2h}^x = \{E, C_{2x}, P, M_x\}$, $C_{2h}^y = \{E, C_{2y}, P, M_y\}$, and $C_{2h}^z = \{E, C_{2z}, P, M_z\}$, respectively. These three choices correspond to the same altermagnetic type, but yield distinct altermagnetic BZs whose spin textures differ only in spatial orientation. Therefore, in orthorhombic systems, choosing different halving subgroups leads to different principal axes and hence to distinct altermagnetic BZs. The same holds for the point group C_{2v} analyzed in the main text: although its three halving subgroups— C_2 , C_{1h}^x , and C_{1h}^y —are genuinely distinct, their extensions under the spinless time-reversal symmetry S coincide with the three orientations of C_{2h} , so the principal axis again points along z , x , or y . To explicitly show this orientational equivalence, Fig. S3 presents the full set of configurations with the x , y , and z axes chosen as the principal axis.

We note that, in orthorhombic systems, A-centered and C-centered lattices are symmetry-equivalent up to a relabeling of the crystallographic axes. Here, a C-centered (A-centered) orthorhombic lattice carries an additional lattice point at the center of the face spanned by the a and b (b and c) axes, located at $(\frac{1}{2}, \frac{1}{2}, 0)$ [$(0, \frac{1}{2}, \frac{1}{2})$] in fractional coordinates. For this reason, Fig. S3 displays only the C-centered case as a representative base-centered orthorhombic lattice. The corresponding A-centered spin-distribution patterns can be obtained directly from the C-centered ones by interchanging the appropriate crystallographic axes.

C. C_{2z} -Protected Spin-Degenerate Lines and Unlocked Nodal Planes in Monoclinic Systems

Here we discuss the distribution of unlocked nodal planes in different types of Brillouin zones for monoclinic systems. Although vertical mirror symmetries are absent and therefore cannot pin the inter-spin nodal planes to fixed high-symmetry planes, the twofold rotational symmetry C_{2z} can still protect opposite-spin degeneracies along certain rotation-invariant momentum

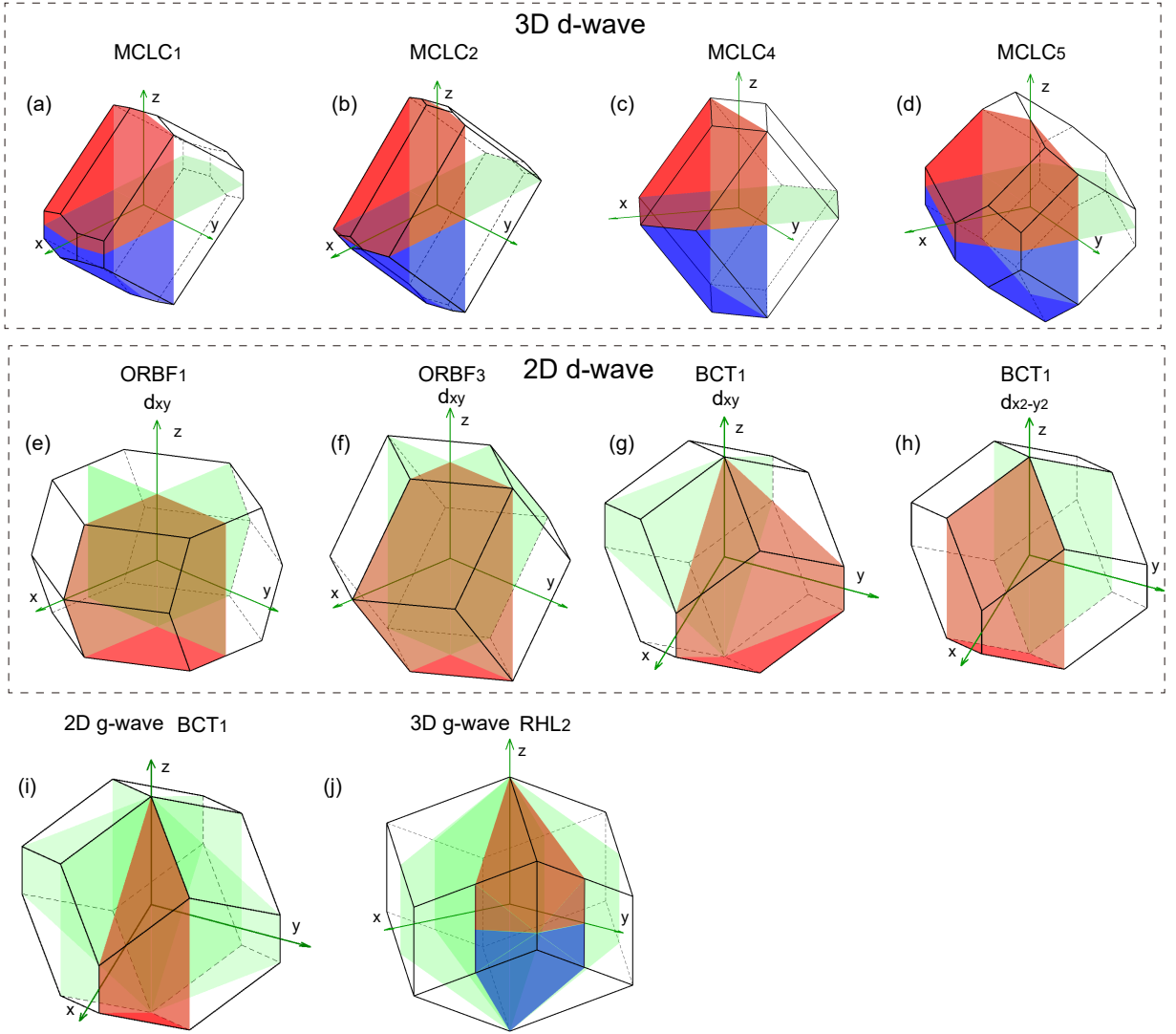


FIG. S2: (color online) Spin-splitting textures in different primitive BZ geometries. (a)–(d) Monoclinic systems with 3D *d*-wave alternating spin textures, shown for the MCLC₁, MCLC₂, MCLC₄, and MCLC₅ BZs, and the main text adopts MCLC₃ as its representative BZ. (e)–(f) Orthorhombic systems with 2D *d*-wave spin textures, shown for the ORCF₁ and ORCF₃ BZs. (g)–(h) Tetragonal systems with 2D *d_{xy}*- and *d_{x2-y2}*-wave spin textures in the BCT₁ BZ. (i) Tetragonal system with 2D *g*-wave spin texture in the BCT₁ BZ. (j) Trigonal system with 3D *g*-wave spin texture in the RHL₂ BZ. Red and blue regions denote spin-up and spin-down states, respectively. Green planes represent symmetry-protected locked nodal planes. In panels (a)–(d), the vertical nodal planes are not fixed by symmetry and are therefore unlocked.

lines. These C_{2z} -protected spin-degenerate lines impose geometric constraints on the distribution of unlocked nodal planes: although the nodal planes themselves may continuously deform, they must pass through these fixed spin-degenerate lines.

In the monoclinic system, the lattice vectors of a simple monoclinic cell can be chosen as

$$\mathbf{a}_1 = (a, 0, 0), \quad \mathbf{a}_2 = (b \cos \beta, b \sin \beta, 0), \quad \mathbf{a}_3 = (0, 0, c), \quad (7)$$

where the $\mathbf{a}_3$ axis is taken as the unique axis. The corresponding reciprocal lattice vectors are

$$\mathbf{b}_1 = \frac{2\pi}{\sin \beta} \left(\frac{1}{a} \sin \beta, -\frac{1}{a} \cos \beta, 0 \right), \quad \mathbf{b}_2 = \frac{2\pi}{\sin \beta} \left(0, \frac{1}{b}, 0 \right), \quad \mathbf{b}_3 = \frac{2\pi}{\sin \beta} \left(0, 0, \frac{1}{c} \sin \beta \right). \quad (8)$$

We first determine the momentum lines that remain invariant under the C_{2z} rotation. A general momentum can be written as

$$\mathbf{k} = k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2 + k_3 \mathbf{b}_3, \quad (9)$$

where k_i are reciprocal-lattice coordinates. Under the C_{2z} rotation, the momentum components perpendicular to the z axis change sign, whereas the k_z component remains unchanged: $C_{2z} \mathbf{k} = -k_1 \mathbf{b}_1 - k_2 \mathbf{b}_2 + k_3 \mathbf{b}_3$. Since crystal momentum is

Crystal system	Bravais lattice	Number of BZs	BZ label	Type of altermagnetism
Triclinic	Simple triclinic (TRI)	4	TRI _{1a,2a,1b,2b}	×
Monoclinic	Simple monoclinic (MCL)	1	MCL	3D d-wave
	C-centered monoclinic (MCLC)	5	MCLC _{1,2,3,4,5}	
Orthorhombic	Simple orthorhombic (ORC)	1	ORC	2D d-wave
	Face-centered orthorhombic (ORCF)	3	ORCF _{1,2,3}	
	Body-centered orthorhombic (ORCI)	1	ORCI	
	C-centered orthorhombic (ORCC)	1	ORCC	
Tetragonal	Simple tetragonal (TET)	1	TET	2D d-wave and g-wave
	Body-centered tetragonal (BCT)	2	BCT _{1,2}	
Trigonal/Hexagonal	Hexagonal (HEX)	1	HEX	3D g-wave and 2D i-wave
	Rhombohedral (RHL)	2	RHL _{1,2}	
Cubic	Simple cubic (CUB)	1	CUB	3D i-wave
	Face-centered cubic (FCC)	1	FCC	
	Body-centered cubic (BCC)	1	BCC	

TABLE S1: Summary of the BZ geometries associated with the 14 Bravais lattices. For each crystal system, the table lists the corresponding Bravais lattice, the number of distinct BZ shapes, the standard BZ labels, and the associated altermagnetic wave type. The symbol $\times$ indicates that the simple triclinic lattice does not support altermagnetic spin splitting.

defined modulo a reciprocal lattice vector, the C_{2z} -invariant condition is

$$C_{2z}\mathbf{k} = \mathbf{k} + n_1\mathbf{b}_1 + n_2\mathbf{b}_2 + n_3\mathbf{b}_3, \quad n_i \in \mathbb{Z}. \quad (10)$$

By comparing the coefficients of $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$, one obtains

$$-2k_1 = n_1, \quad -2k_2 = n_2, \quad 0 = n_3. \quad (11)$$

Thus,

$$k_1 = 0 \text{ or } \frac{1}{2}, \quad k_2 = 0 \text{ or } \frac{1}{2}, \quad (12)$$

while k_3 remains a continuous parameter. Therefore, in the simple monoclinic Brillouin zone there are four C_{2z} -invariant momentum lines along the k_z direction,

$$(k_1, k_2, k_3) = (0, 0, k_3), \quad (\frac{1}{2}, 0, k_3), \quad (\frac{1}{2}, \frac{1}{2}, k_3), \quad (0, \frac{1}{2}, k_3). \quad (13)$$

Substituting these coordinates into the reciprocal lattice vectors gives their Cartesian representations:

$$l_1 = (0, 0, k_z), l_2 = \frac{2\pi}{\sin\beta}(\frac{1}{2a}\sin\beta, -\frac{1}{2a}\cos\beta, k_z), l_3 = \frac{2\pi}{\sin\beta}(\frac{1}{2a}\sin\beta, -\frac{1}{2a}\cos\beta + \frac{1}{2b}, k_z), l_4 = \frac{2\pi}{\sin\beta}(0, \frac{1}{2b}, k_z).$$

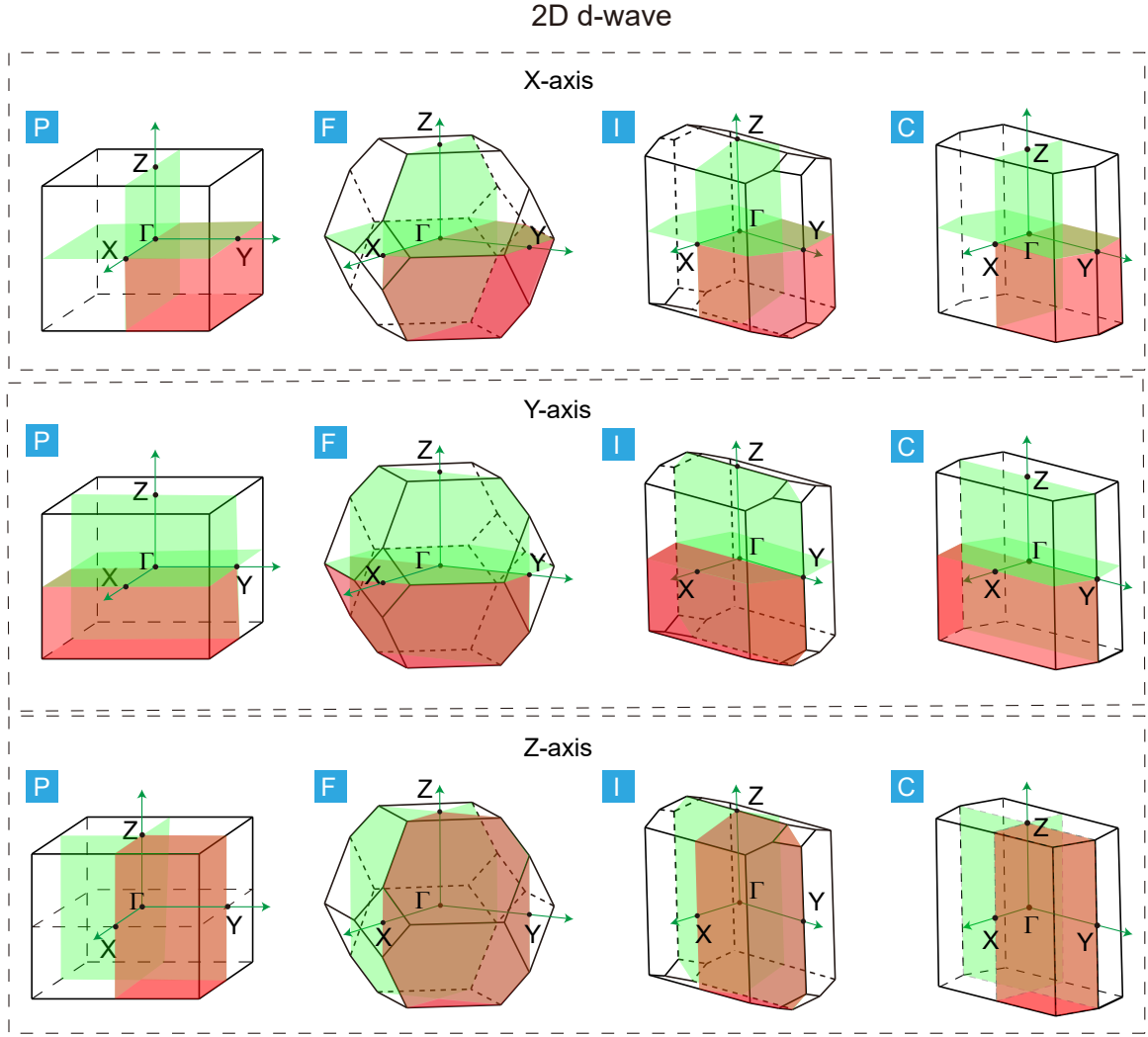


FIG. S3: (color online) Momentum-dependent 2D *d*-wave spin-splitting textures in orthorhombic systems with different choices of the principal axis. The three rows correspond to the *x*-, *y*-, and *z*-axis configurations, respectively. In each row, panels P, F, I, and C show the Brillouin zones of primitive, face-centered, body-centered, and C-centered orthorhombic lattices, respectively. Red and green regions denote opposite spin polarizations, and the nodal planes separating them indicate spin-degenerate manifolds.

These momentum lines are highlighted by green lines in Fig. S4(a).

For a C-centered monoclinic lattice, the primitive lattice vectors can be chosen as

$$\mathbf{a}'_1 = (a, 0, 0), \quad \mathbf{a}'_2 = \left(\frac{1}{2}b \cos \beta, \frac{1}{2}b \sin \beta, -\frac{1}{2}c \right), \quad \mathbf{a}'_3 = \left(\frac{1}{2}b \cos \beta, \frac{1}{2}b \sin \beta, \frac{1}{2}c \right). \quad (14)$$

The corresponding reciprocal lattice vectors are

$$\mathbf{b}'_1 = \frac{2\pi}{\sin \beta} \left(\frac{1}{a} \sin \beta, -\frac{1}{a} \cos \beta, 0 \right), \mathbf{b}'_2 = \frac{2\pi}{\sin \beta} \left(0, \frac{1}{b}, -\frac{1}{c} \sin \beta \right) = \mathbf{b}_2 - \mathbf{b}_3, \mathbf{b}'_3 = \frac{2\pi}{\sin \beta} \left(0, \frac{1}{b}, \frac{1}{c} \sin \beta \right) = \mathbf{b}_2 + \mathbf{b}_3. \quad (15)$$

The momentum can be written as $\mathbf{k} = k'_1 \mathbf{b}'_1 + k'_2 \mathbf{b}'_2 + k'_3 \mathbf{b}'_3$. Because $\mathbf{b}'_2$ and $\mathbf{b}'_3$ contain opposite and identical *z*-components, respectively, they are exchanged under the C_{2z} operation while their transverse components change sign. Equivalently, one may express them in terms of the conventional reciprocal lattice vectors,

$$\mathbf{b}'_2 = \mathbf{b}_2 - \mathbf{b}_3, \quad \mathbf{b}'_3 = \mathbf{b}_2 + \mathbf{b}_3. \quad (16)$$

In this representation, the C_{2z} -invariant condition still requires the transverse momentum after rotation to differ from the original one only by a reciprocal lattice vector. Owing to the reciprocal-lattice identifications of the C-centered primitive cell, part of

the invariant lines in the simple-cell description become equivalent. As a result, only two independent C_{2z} -invariant momentum lines remain: $(0, 0, k'_3)$ and $(\frac{1}{2}, 0, k'_3)$. Their Cartesian coordinates can be written as

$$l_1 = (0, 0, k_z), \quad l_2 = \frac{2\pi}{\sin \beta} \left(\frac{1}{2a} \sin \beta, -\frac{1}{2a} \cos \beta, k_z \right). \quad (17)$$

As shown in Fig. S4(d), these two lines are the C_{2z} -invariant lines along the k_z direction in the Brillouin zone of the C-centered monoclinic primitive cell.

In the altermagnetic phase, the C_{2z} rotation connects sublattices with opposite spin and thus protects spin degeneracies along the C_{2z} -invariant momentum lines. For the simple monoclinic cell, there are four such spin-degenerate lines. Although unlocked nodal planes are not pinned by mirror symmetries, they must pass through these C_{2z} -protected spin-degenerate lines; hence, these lines constrain the global distribution of the spin-degenerate nodal planes.

To illustrate the spin distribution more clearly, we consider a two-dimensional momentum slice at fixed k_z . On this slice, the four C_{2z} -invariant spin-degenerate lines project to four spin-degenerate points, shown as green points in Fig. S4(b). These degenerate points can be connected by spin-degenerate lines, which correspond to the intersections of the unlocked nodal planes with the two-dimensional slice. We then extend the two-dimensional spin distribution shown in the panel of Fig. S4(b) into the full three-dimensional Brillouin zone, obtaining the spin-state distribution shown in Fig. S4(c).

III. ALTERMAGNETIC GROUP CLASSIFICATION AND NON-CENTROSYMMETRIC CANDIDATE MATERIALS

This section provides supplementary details on the symmetry classification of altermagnetic groups and the identification of non-centrosymmetric candidate materials. First, we list all altermagnetic wallpaper groups and altermagnetic space groups from their parent wallpaper and space groups, which satisfy the symmetry requirements for altermagnetism. Second, we describe the first-principles methods used to verify the spin-resolved electronic structures of representative candidates. Third, we present the symmetry analysis and band structures of non-centrosymmetric altermagnetic candidates screened from the MAGNDATA database.

A. Altermagnetic wallpaper and space group

In the main text, we systematically explore the magnetic extensions of the ten 2D point groups to identify those compatible with altermagnetic order—designated as altermagnetic point groups (APGs)—and subsequently derive the associated altermagnetic wallpaper groups (AWGs). Our classification yields 17 distinct AWGs, consisting of 12 centrosymmetric and 5 non-centrosymmetric groups, as summarized in Table S3. The table lists the parent wallpaper group (WG) alongside their corresponding altermagnetic wallpaper group (AWGs). For each AWG, we provide the underlying point group, the halving subgroup $\mathcal{H}$, and the coset representative $\mathcal{R}$ that connects the two spin sublattices. The final column categorizes the symmetry of the resulting momentum-space spin-splitting (e.g., d -, g -, or i -wave character). Throughout the classification tables, mirror operations

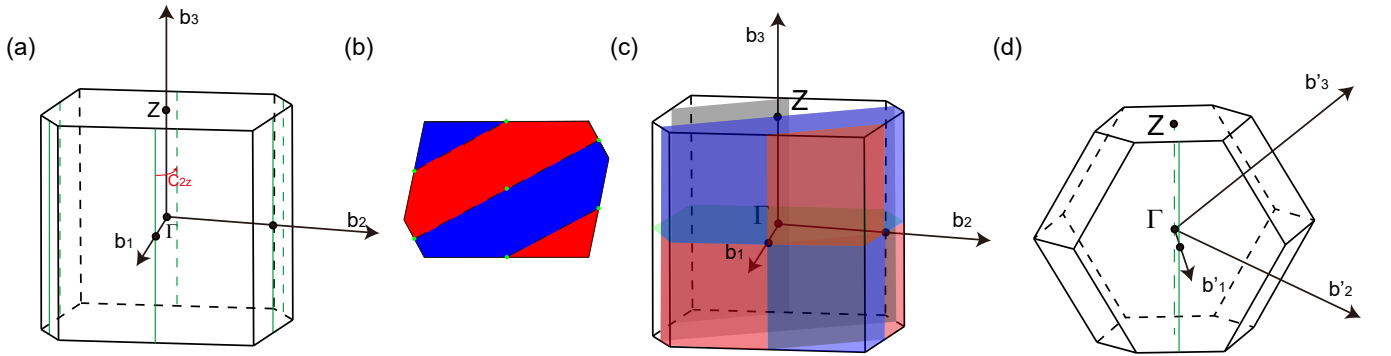


FIG. S4: (color online) Schematic illustration of C_{2z} -protected spin-degenerate lines and unlocked nodal planes in monoclinic Brillouin zones. (a) Brillouin zone of the simple monoclinic cell, where the green lines denote momentum lines invariant under the C_{2z} rotation. (b) Two symmetry-equivalent spin-distribution patterns on a two-dimensional momentum slice at fixed k_z . The green points represent the intersections between the C_{2z} -invariant spin-degenerate lines and the fixed- k_z plane. (c) Three-dimensional spin-state distribution obtained by extending the configuration in (b) throughout the conventional Brillouin zone. (d) Brillouin zone of the C-centered monoclinic cell, where two independent C_{2z} -invariant momentum lines remain. The green lines indicate the C_{2z} -protected spin-degenerate lines.

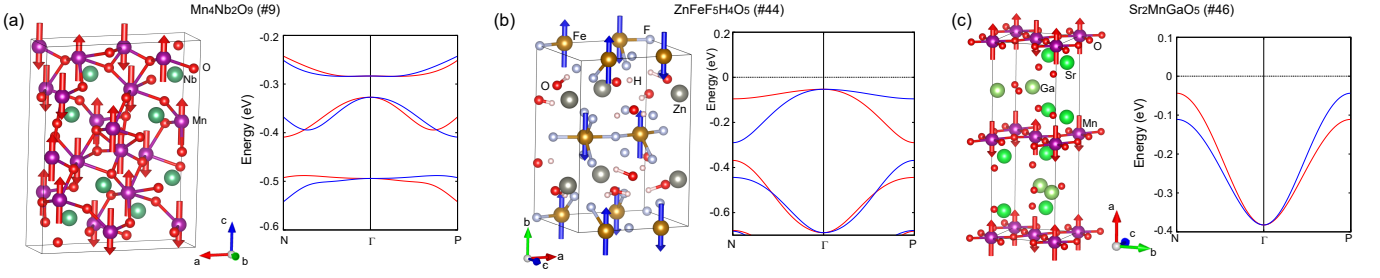


FIG. S5: (color online) Crystal structure with collinear spin arrangement, calculated spin-resolved band structure of non-centrosymmetric candidate materials in monoclinic and orthorhombic crystal systems (a) $\text{Mn}_4\text{Nb}_2\text{O}_9$, (b) $\text{ZnFeF}_5\text{H}_4\text{O}_5$ and (c) $\text{Sr}_2\text{MnGaO}_5$. The two paths $N\text{-}\Gamma\text{-}P$ are connected by the corresponding symmetry, and red and blue lines represent spin up and down bands.

are labeled by their normal axes (e.g., M_x), diagonal mirrors by the axes they exchange (e.g., $M_{x=y}$), or by the Schoenflies classes σ_v and σ_d ; in the wave-type labels, the subscript specifies the harmonic form of the spin splitting (e.g., $d_{xy} \propto k_x k_y$), and the superscript denotes the Bravais-lattice type of the parent group. Groups that do not support altermagnetic order are indicated by a cross ($\times$).

Extending this framework to three dimensions, we analyze the 32 crystallographic point groups to determine their altermagnetic counterparts. We identify 37 distinct APGs, comprising 10 centrosymmetric and 27 non-centrosymmetric cases. By incorporating translational symmetries, these APGs give rise to 422 unique altermagnetic space groups (ASGs), categorized into 160 centrosymmetric and 262 non-centrosymmetric groups, as detailed in Tables S4-S16. For each parent space group, the table lists the corresponding altermagnetic space groups (ASGs). We provide the decomposition into the halving subgroup ($\mathcal{H}$) and the coset representative ($\mathcal{R}$) that connects the two spin sublattices. The Type column indicates the symmetry of the momentum-space spin-splitting (e.g., $3D\ d^P$ -wave), while the final column denotes the presence of inversion symmetry. Groups marked with ' $\times$ ' are incompatible with altermagnetic order.

B. Computational method

Our electronic-structure calculations were performed using the Vienna *ab initio* Simulation Package (VASP)¹ within the projector augmented-wave (PAW) method². The exchange-correlation interaction was treated using the Perdew–Burke–Ernzerhof (PBE) generalized-gradient approximation³. The plane-wave kinetic-energy cutoff was chosen to be sufficiently large to ensure the convergence of the total energy and band structure. The Brillouin zone was sampled using Monkhorst–Pack (k)-point meshes⁴. Spin-orbit coupling was not included in the calculations, in order to focus on the spin-group-symmetry-protected altermagnetic band structures. Experimental lattice parameters and atomic positions were adopted for all candidate materials^{7–18}.

C. Non-centrosymmetric candidate materials

In the main text, we employed the MAGNDATA database^{5,6} on the Bilbao Crystallographic Server to identify non-centrosymmetric candidates for hosting altermagnetic order. In the monoclinic and orthorhombic crystal systems, we identified three candidates: $\text{Mn}_4\text{Nb}_2\text{O}_9$ ⁷, $\text{ZnFeF}_5\text{H}_4\text{O}_5$ ⁸, and $\text{Sr}_2\text{MnGaO}_5$ ⁹. In the tetragonal system, the candidates include $\text{SrMn}_2\text{V}_2\text{O}_8$ ¹⁰, $\text{BaMn}_2\text{V}_2\text{O}_8$ ¹¹, CsCoF_4 ¹², and CuFeS_2 ¹³. In the trigonal and hexagonal systems, we found five candidates: PbNiO_3 ¹⁴, MnTiO_3 ¹⁵, VNb_2S_6 ¹⁶, $\text{Co}_2\text{Mo}_3\text{O}_8$ ¹⁷, and $\text{Fe}_2\text{Mo}_3\text{O}_8$ ¹⁸. Table S2 shows the symmetry analysis of these candidates, which gives the altermagnetic space group of each candidate. Figs. S5 and S6 show crystal structures with collinear spin arrangement, and spin-resolved band structures of these candidate materials, which exhibit spin-splitting. In the following, we give detailed symmetry analysis and band structures for these candidate materials.

- $\text{Mn}_4\text{Nb}_2\text{O}_9$: This compound crystallizes in the base-centered monoclinic space group Cc (No. 9). It features the nonsym-morphic symmetry $\tilde{M}_y = \{M_y | 0\ 0\ 1/2\}$, which links sublattices with opposite spins, thereby enforcing opposite-spin degeneracy in the xz plane. By incorporating the non-spatial global symmetry S , an additional effective rotational symmetry $\tilde{C}_{2y} = \tilde{M}_y S$ can emerge to connect opposite-spin sites, which protects an unlocked opposite-spin degenerate plane. The interplay of these two planes results in a prominent fourfold symmetry in the spin-splitting distribution across the BZ, identifying $\text{Mn}_4\text{Nb}_2\text{O}_9$ as a $3D\ d$ -wave altermagnet. Fig. S5(a) shows the spin-split bands along any path $N\text{-}\Gamma\text{-}P$, where $N\text{-}\Gamma$ and $\Gamma\text{-}P$ are related by $\tilde{C}_{2y}$. Under the action of $\tilde{C}_{2y}$, spin-up bands transform into spin-down bands.

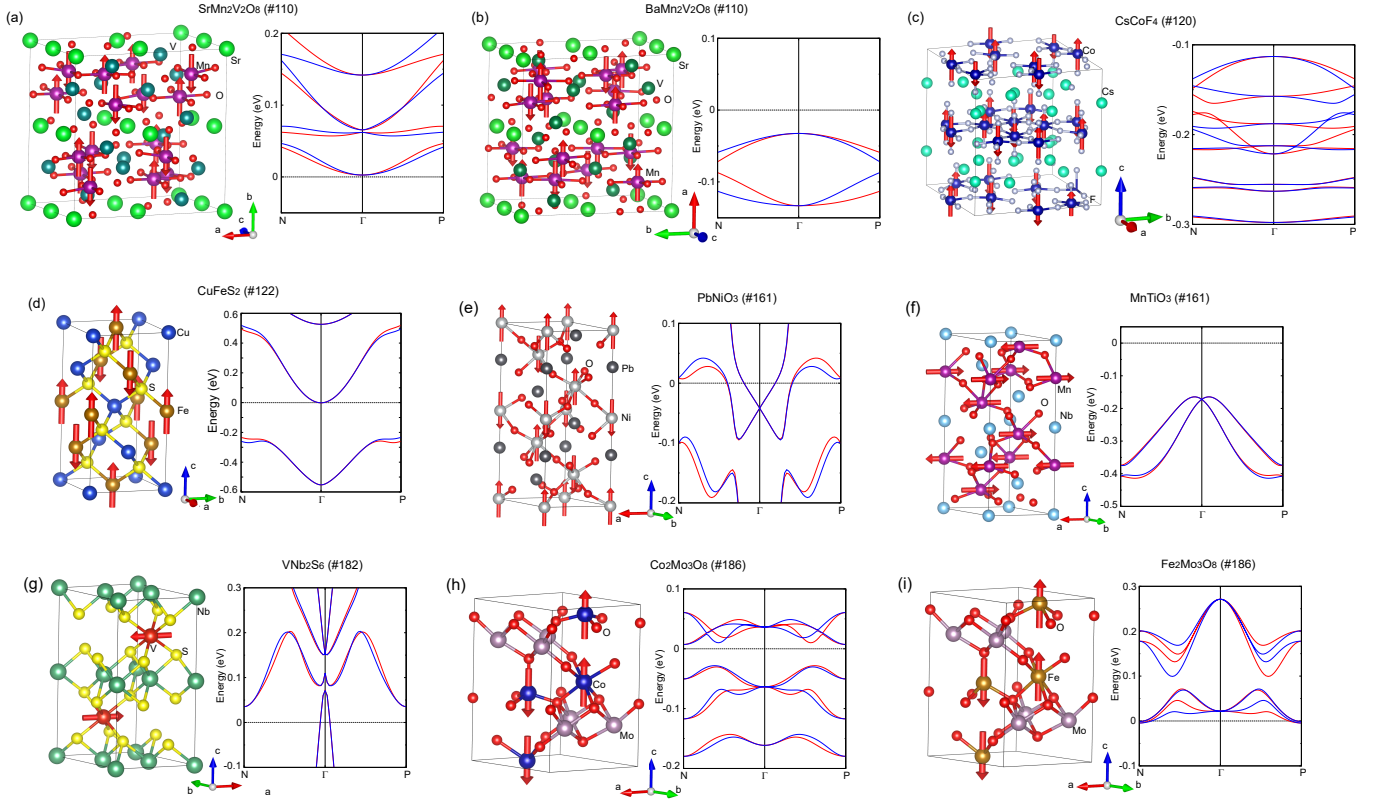


FIG. S6: (color online) Crystal structure with collinear spin arrangement, calculated spin-resolved band structure of non-centrosymmetric candidate materials in tetragonal, trigonal and hexagonal crystal systems (a)-(d) $\text{SrMn}_2\text{V}_2\text{O}_8$ ¹⁰, $\text{BaMn}_2\text{V}_2\text{O}_8$ ¹¹, CsCoF_4 ¹² and CuFeS_2 ¹³. (e)-(i) PbNiO_3 ¹⁴, MnTiO_3 ¹⁵, VNb_2S_6 ¹⁶, $\text{Co}_2\text{Mo}_3\text{O}_8$ ¹⁷ and $\text{Fe}_2\text{Mo}_3\text{O}_8$ ¹⁸. The two paths $N\text{-}\Gamma\text{-}P$ are connected by the symmetry, and red and blue lines represent spin up and down bands.

- $\text{ZnFeF}_5\text{H}_4\text{O}_5$: This material crystallizes in the space group $Imm2$ (No.44). The associated point group C_{2v} contains only the halving subgroup $C_{1h}^y = \{E, M_y\}$, which connects the same-spin sublattices, as shown in Fig. S5(b). Conversely, the coset symmetry M_x and effective symmetry $\tilde{M}_z = C_{2z}S$ link sublattices with opposite spin, enforcing opposite-spin degeneracy in the yz and xy planes. Consequently, $\text{ZnFeF}_5\text{H}_4\text{O}_5$ is classified as a 2D d -wave altermagnet. As shown in Fig. S5(b), spin-up and spin-down bands along the $N\text{-}\Gamma\text{-}P$ path are related by M_x , which transforms spin-up states to spin-down states.
- $\text{Sr}_2\text{MnGaO}_5$: This compound crystallizes in the nonsymmorphic space group $Ima2$ (No. 46), featuring the operations C_{2z} , $\tilde{M}_y = \{M_y|1/2\ 0\ 0\}$, and $\tilde{M}_x = \{M_x|1/2\ 0\ 0\}$. The associated point group C_{2v} contains only the halving subgroup C_{2z} , which connects the same-spin sublattices, as shown in Fig. S5(c). The coset symmetries $\tilde{M}_y$ and $\tilde{M}_x$ link sublattices with opposite spin, enforcing opposite-spin degeneracy in the xz and yz planes. Thus, $\text{Sr}_2\text{MnGaO}_5$ behaves as a 2D d_{xy} -wave altermagnet. In Fig. S5(c), the spin-split bands along the $N\text{-}\Gamma\text{-}P$ path demonstrate that the bands are mapped via $\tilde{M}_x$, which serves as the spin-flip operation.
- $\text{SrMn}_2\text{V}_2\text{O}_8$ and $\text{BaMn}_2\text{V}_2\text{O}_8$: Both compounds crystallize in the nonsymmorphic space group $I4_1cd$ (No. 110), featuring nonsymmorphic symmetries $\tilde{C}_{4z} = \{C_{4z}|0\ 1/2\ 1/4\}$, $\tilde{C}_{2z} = \{C_{2z}|1/2\ 1/2\ 1/2\}$, and $\tilde{M}_y = \{M_y|0\ 0\ 1/2\}$. Their associated point group C_{4v} contains the halving subgroup $C_{2v} = \{E, \tilde{C}_{2z}, \tilde{M}_{110}, \tilde{M}_{1-10}\}$, which preserves same-sublattice spin alignment [Figs. S6(a) and (b)]. The remaining coset elements $\tilde{M}_y$ and M_x link sublattices with opposite spins, enforcing opposite-spin degeneracy along the BZ boundaries. Thus, these compounds are identified as 2D d_{xy} -wave altermagnets. Spin-resolved band structures confirm that these symmetry operations act as spin-flip operators, mapping spin-up states to spin-down states.
- CsCoF_4 : This material crystallizes in the nonsymmorphic space group $I\bar{4}c2$ (No.120), including operations S_{4z} , C_{2z} , and $\tilde{M}_y = \{M_y|0\ 0\ 1/2\}$. The point group D_{2d} contains a halving subgroup $D_2 = \{E, \tilde{C}_{2z}, C_{110}, C_{1-10}\}$ that connects sublattices with the same spin [Fig. S6(c)]. The coset symmetries enforce inter-spin degeneracy in the xy and yz planes,

classifying CsCoF_4 as a 2D d_{xy} -wave altermagnet. The band structure in Fig. S6(c) exhibits spin splitting, where the spin-flip operation is mediated by the mirror symmetry.

- CuFeS_2 : This compound crystallizes in the nonsymmorphic space group $I\bar{4}2d$ (No.122), featuring symmetries S_{4z} , C_{2z} , and $\tilde{C}_{2x} = \{C_{2x}|1/2\ 0\ 3/4\}$. Its point group D_{2d} contains the halving subgroup S_4 that preserves intra-sublattice spin alignment [Fig. S6(d)]. The mirror and rotational operations connect sublattices with opposite spins, enforcing opposite-spin degenerate planes. Consequently, CuFeS_2 acts as a 2D g -wave altermagnet, with spin-split bands satisfying the required symmetry-enforced mappings, as shown in Fig. S6(d).
- PbNiO_3 and MnTiO_3 : These compounds crystallize in the nonsymmorphic space group $R3c$ (No.161), including operations C_{3z} and $\tilde{M}_x = \{M_x|0\ 0\ 1/2\}$. The point group C_{3v} contains a halving subgroup C_3 that connects sublattices with the same spin [Figs. S6(e) and (f)]. The mirror symmetries link sublattices with opposite spins, protecting spin-degenerate planes. These materials are classified as 3D g -wave altermagnets, exhibiting characteristic spin splitting in Figs. S6(e) and (f).
- VNb_2S_6 : This compound crystallizes in the nonsymmorphic space group $P6_322$ (No.182), featuring operations $\tilde{C}_{6z} = \{C_{6z}|0\ 0\ 1/2\}$ and C_{3z} . Its point group D_6 contains a halving subgroup D_3 that connects sublattices with the same spin [Fig. S6(g)]. The coset symmetry connects sublattices with opposite spins, enforcing spin-degenerate planes. As a result, VNb_2S_6 behaves as a 3D g -wave altermagnet with characteristic spin splitting.
- $\text{Co}_2\text{Mo}_3\text{O}_8$ and $\text{Fe}_2\text{Mo}_3\text{O}_8$: These compounds crystallize in the nonsymmorphic space group $P6_3mc$ (No. 186), including operations C_{6z} and C_{3z} . The point group C_{6v} contains a halving subgroup C_{3v} that preserves same-sublattice spin alignment [Figs. S6(h) and (i)]. The coset symmetry operations connecting the opposite-spin sublattices enforce inter-spin degeneracy, classifying these materials as 3D g -wave altermagnets. Their electronic structures demonstrate prominent spin splitting, as shown in Figs. S6(h) and (i).

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Crystal system	Material	Point group	Subgroup	Coset	Space group	ASG	Type
Monoclinic	$\text{Mn}_4\text{Nb}_2\text{O}_9$	C_{1h}^y	E	M_y	#9 (Cc)	9.3.47 (Cc')	3D d^P -wave
Orthorhombic	$\text{Y}_2\text{Cu}_2\text{O}_5$	C_{2v}	C_{1h}^y	C_{2z}	#33 ($Pna2_1$)	33.3.228 ($Pn'a2'_1$)	2D d_{xz}^P -wave
	$\text{ZnFeF}_5\text{H}_4\text{O}_5$	C_{2v}	C_{1h}^y	C_{2z}	#44 ($Imm2$)	44.3.326 ($Im'm2'$)	2D d_{xz}^I -wave
	$\text{Sr}_2\text{MnGaO}_5$	C_{2v}	C_{2z}	M_y	#46 ($Ima2$)	46.5.342 ($Im'a'2$)	2D d_{xy}^I -wave
Tetragonal	$\text{SrMn}_2\text{V}_2\text{O}_8$	C_{4v}	C_{2v}	C_{4z}	#110 ($I4_1cd$)	110.3.908 ($I4'_1c'd$)	2D d_{xy}^I -wave
	$\text{BaMn}_2\text{V}_2\text{O}_8$	C_{4v}	C_{2v}	C_{4z}	#110 ($I4_1cd$)	110.3.908 ($I4'_1c'd$)	
	CsCoF_4	D_{2d}	D_2	S_{4z}	#120 ($I\bar{4}c2$)	120.3.980 ($I\bar{4}'c'2$)	
	CuFeS_2	D_{2d}	S_4	C_{2x}	#122 ($I\bar{4}2d$)	122.5.998 ($I\bar{4}2'd'$)	2D g -wave
Trigonal	PbNiO_3	C_{3v}	C_3	σ'	#161 ($R3c$)	161.3.1302 ($R3c'$)	3D g -wave
	MnTiO_3	C_{3v}	C_3	σ'	#161 ($R3c$)	161.3.1302 ($R3c'$)	
Hexagonal	VNb_2S_6	D_6	D_3	C_{6z}	#182 ($P6_322$)	182.3.1412 ($P6'_32'2$)	3D g -wave
	$\text{Co}_2\text{Mo}_3\text{O}_8$	C_{6v}	C_{3v}	σ_v	#186 ($P6_3mc$)	186.3.1436 ($P6'_3mc'$)	
	$\text{Fe}_2\text{Mo}_3\text{O}_8$	C_{6v}	C_{3v}	σ_v	#186 ($P6_3mc$)	186.3.1436 ($P6'_3mc'$)	

TABLE S2: Classification of non-centrosymmetric candidate materials. The table maps representative materials to their corresponding crystal systems, point groups, and altermagnetic space groups (ASGs).

WG	AWG	Point group	Subgroup	Coset	Type
#1 (P1)	$\times$	C_1	$\times$	$\times$	$\times$
#2 (P2)	$\times$	C_2	$\times$	$\times$	$\times$
#3 (P1m1)	3.3.10 (P1m'1)	C_{1h}	C_1	M_x	$d_{x^2-y^2}^P$ -wave
#4 (P1g1)	4.3.17 (P1g'1)				$d_{x^2-y^2}^C$ -wave
#5 (C1m1)	5.3.21 (C1m'1)				
#6 (P2mm)	6.3.26 (P2m'm')	C_{2v}	C_2	M_x	$d_{x^2-y^2}^P$ -wave
#7 (P2mg)	7.3.33 (P2m'g')				$d_{x^2-y^2}^C$ -wave
#8 (P2gg)	8.3.40 (P2g'g')				
#9 (C2mm)	9.3.44 (C2m'm')				
#10 (P4)	10.3.51 (P4')	C_4	C_2	C_{4z}	d_{mix}^S -wave
#11 (P4mm)	11.3.55 (P4m'm')	C_{4v}	C_4	$M_{x=y}/M_x$	g -wave
	11.4.56 (P4'mm')	C_{4v}	C_{2v}	C_{4z}	d_{xy}^S -wave
	11.5.57 (P4'm'm)			C_{4z}	$d_{x^2-y^2}^S$ -wave
#12 (P4gm)	12.3.62 (P4g'm')	C_{4v}	C_4	$M_{x=y}/M_x$	g -wave
	12.4.63 (P4'gm')	C_{4v}	C_{2v}	C_{4z}	d_{xy}^S -wave
	12.5.64 (P4'g'm)			C_{4z}	$d_{x^2-y^2}^S$ -wave
#13 (P3)	$\times$	C_3	$\times$	$\times$	$\times$
#14 (P3m1)	14.3.69 (P3m'1)	C_{3v}	C_3	M_y	i -wave
#15 (P31m)	15.3.72 (P31m')			M_x	i -wave
#16 (P6)	$\times$	C_6	$\times$	$\times$	$\times$
#17 (P6mm)	17.3.78 (P6m'm')	C_{6v}	C_6	M_x	i -wave

TABLE S3: Classification of altermagnetic wallpaper groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#1 (P1)	$\times$	C_1	$\times$	$\times$	$\times$	No
#2 ($P\bar{1}$)	$\times$	S_2	$\times$	$\times$	$\times$	Yes
#3 (P2)	3.3.10 ($P2'$)	C_2	C_1	C_{2z}	3D d^P -wave	No
#4 ($P2_1$)	4.3.17 ($P2'_1$)				3D d^C -wave	
#5 (C2)	5.3.21 ($C2'$)					
#6 (Pm)	6.3.27 (Pm')	C_{1h}	C_1	M_x	3D d^P -wave	No
#7 (Pc)	7.3.34 (Pc')				3D d^C -wave	
#8 (Cm)	8.3.40 (Cm')					
#9 (Cc)	9.3.47 (Cc')					
#10 (P2/m)	10.5.53 ($P2'/m'$)	C_{2h}	S_2	C_{2z}, M_z	3D d^P -wave	Yes
#11 ($P2_1/m$)	11.5.63 ($P2'_1/m'$)				3D d^C -wave	
#12 (C2/m)	12.5.70 ($C2'/m'$)				3D d^P -wave	
#13 ($P2_1/c$)	13.5.81 ($P2'/c'$)				3D d^C -wave	
#14 ($P2_1/c$)	14.5.90 ($P2'_1/c'$)					
#15 (C2/c)	15.5.96 ($C2'/c'$)					
#16 (P222)	16.3.101 ($P2'2'2$)	D_2	C_2	C_{2x}, C_{2y}	2D d^P_{xy} -wave	No
#17 ($P222_1$)	17.3.108 ($P2'2'2_1$)		C_2	C_{2x}, C_{2y}	2D d^P_{xy} -wave	
	17.4.109 ($P22'2'_1$)		C_2	C_{2y}, C_{2z}	2D d^P_{yz} -wave	
#18 ($P2_12_12$)	18.3.115 ($P2'_12'_12$)		C_2	C_{2x}, C_{2y}	2D d^P_{xy} -wave	
	18.4.116 ($P2_12'_12'$)		C_2	C_{2y}, C_{2z}	2D d^P_{yz} -wave	
#19 ($P2_12_12_1$)	19.3.121 ($P2'_12'_12_1$)		C_2	C_{2x}, C_{2y}	2D d^P_{xy} -wave	
#20 (C222 ₁)	20.3.124 ($C2'2'2_1$)		C_2	C_{2x}, C_{2y}	2D d^C_{xy} -wave	
	20.4.125 ($C22'2'_1$)		C_2	C_{2y}, C_{2z}	2D d^C_{yz} -wave	
#21 (C222)	21.3.131 ($C2'2'2$)		C_2	C_{2x}, C_{2y}	2D d^C_{xy} -wave	
	21.4.132 ($C22'2'$)		C_2	C_{2y}, C_{2z}	2D d^C_{yz} -wave	
#22 (F222)	22.3.142 ($F2'2'2$)		C_2	C_{2x}, C_{2y}	2D d^F_{xy} -wave	
#23 (I222)	23.3.147 ($I2'2'2$)		C_2	C_{2x}, C_{2y}	2D d^I_{xy} -wave	
#24 ($I2_12_12_1$)	24.3.152 ($I2'_12'_12_1$)		C_2	C_{2x}, C_{2y}	2D d^I_{xy} -wave	

TABLE S4: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#25 (Pmm2)	25.3.157 (Pm'm2')	C_{2v}	C_{1h}	C_{2z}	2D d_{xz}^P -wave	No
	25.4.158 (Pm'm'2')		C_2	M_y, M_x	2D d_{xy}^P -wave	
#26 (Pmc2 ₁)	26.3.170 (Pm'c2' ₁)		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	26.4.171 (Pmc'2' ₁)		C_{1h}	C_{2z}	2D d_{yz}^P -wave	
	26.5.172 (Pm'c'2 ₁)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#27 (Pcc2)	27.3.180 (Pc'c2')		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	27.4.181 (Pc'c'2)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#28 (Pma2)	28.3.187 (Pm'a2')		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	28.4.188 (Pma'2')		C_{1h}	C_{2z}	2D d_{yz}^P -wave	
	28.5.189 (Pm'a'2)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#29 (Pca2 ₁)	29.3.200 (Pc'a2' ₁)		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	29.4.201 (Pca'2' ₁)		C_{1h}	C_{2z}	2D d_{yz}^P -wave	
	29.5.202 (Pc'a'2 ₁)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#30 (Pnc2)	30.3.207 (Pn'c2')		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	30.4.208 (Pnc'2')		C_{1h}	C_{2z}	2D d_{yz}^P -wave	
	30.5.209 (Pn'c'2)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#31 (Pmn2 ₁)	31.3.214 (Pm'n2' ₁)		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	31.4.215 (Pmn'2' ₁)		C_{1h}	C_{2z}	2D d_{yz}^P -wave	
	31.5.216 (Pm'n'2 ₁)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#32 (Pba2)	32.3.221 (Pb'a2')		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	32.4.222 (Pb'a'2)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#33 (Pna2 ₁)	33.3.228 (Pn'a2' ₁)		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	33.4.229 (Pna'2' ₁)		C_{1h}	C_{2z}	2D d_{yz}^P -wave	
	33.5.230 (Pn'a'2 ₁)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#34 (Pnn2)	34.3.233 (Pn'n2')		C_{1h}	C_{2z}	2D d_{xz}^P -wave	
	34.4.234 (Pn'n'2)		C_2	M_y, M_x	2D d_{xy}^P -wave	
#35 (Cmm2)	35.3.238 (Cm'm2')		C_{1h}	C_{2z}	2D d_{xz}^C -wave	
	35.4.239 (Cm'm'2)		C_2	M_y, M_x	2D d_{xy}^C -wave	
#36 (Cmc2 ₁)	36.3.251 (Cm'c2' ₁)		C_{1h}	C_{2z}	2D d_{xz}^C -wave	
	36.4.252 (Cmc'2' ₁)		C_{1h}	C_{2z}	2D d_{yz}^C -wave	
	36.4.252 (Cm'c'2 ₁)		C_2	M_y, M_x	2D d_{xy}^C -wave	
#37 (Ccc2)	37.3.260 (Cc'c2')		C_{1h}	C_{2z}	2D d_{xz}^C -wave	
	37.4.261 (Cc'c'2)		C_2	M_y, M_x	2D d_{xy}^C -wave	

TABLE S5: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#38 (Amm2)	38.3.267 (Am'm2')	C_{2v}	C_{1h}	C_{2z}	2D d_{xz}^A -wave	No
	38.4.268 (Amm'2')		C_{1h}	C_{2z}	2D d_{yz}^A -wave	
	38.5.269 (Am'm'2)		C_2	M_y, M_x	2D d_{xy}^A -wave	
#39 (Abm2)	39.3.280 (Ab'm2')		C_{1h}	C_{2z}	2D d_{xz}^A -wave	
	39.4.281 (Abm'2')		C_{1h}	C_{2z}	2D d_{yz}^A -wave	
	39.5.282 (Ab'm'2)		C_2	M_y, M_x	2D d_{xy}^A -wave	
#40 (Ama2)	40.3.293 (Am'a2')		C_{1h}	C_{2z}	2D d_{xz}^A -wave	
	40.4.294 (Ama'2')		C_{1h}	C_{2z}	2D d_{yz}^A -wave	
	40.5.295 (Am'a'2)		C_2	M_y, M_x	2D d_{xy}^A -wave	
#41 (Aba2)	41.3.302 (Ab'a2')		C_{1h}	C_{2z}	2D d_{xz}^A -wave	
	41.4.303 (Aba'2')		C_{1h}	C_{2z}	2D d_{yz}^A -wave	
	41.5.304 (Ab'a'2)		C_2	M_y, M_x	2D d_{xy}^A -wave	
#42 (Fmm2)	42.3.311 (Fm'm2')		C_{1h}	C_{2z}	2D d_{xz}^F -wave	
	42.4.312 (Fm'm'2)		C_2	M_y, M_x	2D d_{yz}^F -wave	
#43 (Fdd2)	43.3.322 (Fd'd2')		C_{1h}	C_{2z}	2D d_{xz}^F -wave	
	43.4.323 (Fd'd'2)		C_2	M_y, M_x	2D d_{yz}^F -wave	
#44 (Imm2)	44.3.326 (Im'm2')		C_{1h}	C_{2z}	2D d_{xz}^I -wave	
	44.4.326 (Im'm'2)		C_2	M_y, M_x	2D d_{yz}^I -wave	
#45 (Iba2)	45.3.333 (Ib'a2')		C_{1h}	C_{2z}	2D d_{xz}^I -wave	
	45.4.334 (Ib'a'2)		C_2	M_y, M_x	2D d_{yz}^I -wave	
#46 (Ima2)	46.3.340 (Im'a2')		C_{1h}	C_{2z}	2D d_{xz}^I -wave	
	46.4.341 (Ima'2')		C_{1h}	C_{2z}	2D d_{yz}^I -wave	
	46.5.342 (Im'a'2)		C_2	M_y, M_x	2D d_{xy}^I -wave	
#47 (Pmmm)	47.4.350 (Pm'm'm)	D_{2h}	C_{2h}	M_y, M_x	2D d_{xy}^P -wave	Yes
#48 (Pnnn)	48.4.361 (Pn'n'n)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
#49 (Pccm)	49.5.368 (Pc'c'm)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	49.6.369 (Pc'cm')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#50 (Pban)	50.5.381 (Pb'a'n)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	50.6.382 (Pb'an')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	

TABLE S6: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#51 (Pmma)	51.6.392 (Pm'm'a)	D_{2h}	C_{2h}	M_y, M_x	2D d_{xy}^P -wave	Yes
	51.7.393 (Pmm'a')		C_{2h}	M_x, M_z	2D d_{yz}^P -wave	
	51.8.394 (Pm'ma')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#52 (Pnna)	52.6.411 (Pn'n'a)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	52.7.412 (Pnn'a')		C_{2h}	M_x, M_z	2D d_{yz}^P -wave	
	52.8.413 (Pn'na')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#53 (Pmna)	53.6.420 (Pm'n'a)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	53.7.421 (Pmn'a')		C_{2h}	M_y, M_z	2D d_{yz}^P -wave	
	53.8.422 (Pm'na')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#54 (Pcca)	54.6.433 (Pc'c'a)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	54.7.434 (Pcc'a')		C_{2h}	M_y, M_z	2D d_{yz}^P -wave	
	54.8.435 (Pc'ca')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#55 (Pbam)	55.5.445 (Pb'a'm)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	55.6.446 (Pb'am')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#56 (Pccn)	56.5.455 (Pc'c'n)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	56.6.456 (Pc'cn')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#57 (Pbcm)	57.6.463 (Pb'c'm)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	57.7.464 (Pbc'm')		C_{2h}	M_y, M_z	2D d_{yz}^P -wave	
	57.8.465 (Pb'cm')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#58 (Pnnm)	58.5.475 (Pn'n'm)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	58.6.476 (Pn'nm')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#59 (Pmmn)	59.5.482 (Pm'm'n)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	59.6.483 (Pm'mn')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#60 (Pbcn)	60.6.493 (Pb'c'n)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	60.7.494 (Pbc'n')		C_{2h}	M_y, M_z	2D d_{yz}^P -wave	
	60.8.495 (Pb'cn')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#61 (Pbca)	61.4.500 (Pb'c'a)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
#62 (Pnma)	62.6.507 (Pn'm'a)		C_{2h}	M_y, M_x	2D d_{xy}^P -wave	
	62.7.508 (Pnm'a')		C_{2h}	M_y, M_z	2D d_{yz}^P -wave	
	62.8.509 (Pn'ma')		C_{2h}	M_x, M_z	2D d_{xz}^P -wave	
#63 (Cmcm)	63.6.516 (Cm'c'm)		C_{2h}	M_y, M_x	2D d_{xy}^C -wave	
	63.7.517 (Cmc'm')		C_{2h}	M_y, M_z	2D d_{yz}^C -wave	
	63.8.518 (Cm'cm')		C_{2h}	M_x, M_z	2D d_{xz}^C -wave	

TABLE S7: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#64 (Cmca)	64.6.533 (Cm'c'a)	D_{2h}	C_{2h}	M_y, M_x	2D d_{xy}^C -wave	Yes
	64.7.534 (Cmc'a')		C_{2h}	M_y, M_z	2D d_{yz}^C -wave	
	64.8.535 (Cm'ca')		C_{2h}	M_x, M_z	2D d_{xz}^C -wave	
#65 (Cmmm)	65.5.549 (Cm'm'm)		C_{2h}	M_y, M_x	2D d_{xy}^C -wave	
	65.6.550 (Cmm'm')		C_{2h}	M_y, M_z	2D d_{yz}^C -wave	
#66 (Cccm)	66.5.568 (Cc'c'm)		C_{2h}	M_y, M_x	2D d_{xy}^C -wave	
	66.6.569 (Ccc'm')		C_{2h}	M_y, M_z	2D d_{yz}^C -wave	
#67 (Cmma)	67.5.581 (Cm'm'a)		C_{2h}	M_y, M_x	2D d_{xy}^C -wave	
	67.6.582 (Cmm'a')		C_{2h}	M_y, M_z	2D d_{yz}^C -wave	
#68 (Ccca)	68.5.598 (Cc'c'a)		C_{2h}	M_y, M_x	2D d_{xy}^C -wave	
	68.6.599 (Ccc'a')		C_{2h}	M_y, M_z	2D d_{yz}^C -wave	
#69 (Fmmm)	69.4.608 (Fm'm'm)		C_{2h}	M_y, M_x	2D d_{xy}^F -wave	
#70 (Fddd)	70.4.619 (Fd'd'd)		C_{2h}	M_y, M_x	2D d_{xy}^F -wave	
#71 (Immm)	71.4.624 (Im'm'm)		C_{2h}	M_y, M_x	2D d_{xy}^I -wave	
#72 (Ibam)	72.5.634 (Ib'a'm)		C_{2h}	M_y, M_x	2D d_{xy}^I -wave	
	72.6.635 (Iba'm')		C_{2h}	M_y, M_z	2D d_{yz}^I -wave	
#73 (Ibca)	73.4.646 (Ib'c'a)		C_{2h}	M_y, M_x	2D d_{xy}^I -wave	
#74 (Imma)	74.5.654 (Im'm'a)		C_{2h}	M_y, M_x	2D d_{xy}^I -wave	
	74.6.655 (Imm'a')		C_{2h}	M_y, M_z	2D d_{yz}^I -wave	
#75 (P4)	75.3.663 (P4')	C_4	C_2	C_{4z}, C_{4z}^{-1}	2D d_{mix}^P -wave	No
#76 (P4 ₁)	76.3.670 (P4' ₁)					
#77 (P4 ₂)	77.3.674 (P4' ₂)					
#78 (P4 ₃)	78.3.681 (P4' ₃)					
#79 (I4)	79.3.685 (I4')				2D d_{mix}^I -wave	
#80 (I4 ₁)	80.3.690 (I4' ₁)					
#81 (P $\bar{4}$)	81.3.695 (P $\bar{4}$ ')	S_4	C_2	S_{4z}, S_{4z}^{-1}	2D d_{mix}^P -wave	No
#82 (I $\bar{4}$)	82.3.701 (I $\bar{4}$ ')				2D d_{mix}^I -wave	
#83 (P4/m)	83.3.705 (P4'/m)	C_{4h}	C_{2h}	C_{4z}, C_{4z}^{-1} S_{4z}, S_{4z}^{-1}	2D d_{mix}^P -wave	Yes
#84 (P4 ₂ /m)	84.3.715 (P4' ₂ /m)					
#85 (P4/n)	85.3.722 (P4'/n)					
#86 (P4 ₂ /n)	86.3.729 (P4' ₂ /n)					
#87 (I4/m)	87.3.735 (I4'/m)				2D d_{mix}^I -wave	
#88 (I4 ₁ /a)	88.3.744 (I4' ₁ /a)					

TABLE S8: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#89 (P422)	89.3.749 (P4'22')	D_4	D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	No
	89.5.751 (P4'2'2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	89.4.750 (P42'2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#90 (P42 ₁ 2)	90.3.759 (P4'2 ₁ 2')		D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	90.5.761 (P4'2' ₁ 2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	90.4.760 (P42' ₁ 2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#91 (P4 ₁ 22)	91.3.766 (P4' ₁ 22')		D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	91.5.768 (P4' ₁ 2'2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	91.4.767 (P4 ₁ 2'2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#92 (P4 ₁ 2 ₁ 2)	92.3.773 (P4' ₁ 2 ₁ 2')		D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	92.5.775 (P4' ₁ 2' ₁ 2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	92.4.774 (P4 ₁ 2' ₁ 2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#93 (P4 ₂ 22)	93.3.778 (P4' ₂ 22')		D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	93.5.780 (P4' ₂ 2'2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	93.4.779 (P4 ₂ 2'2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#94 (P4 ₂ 2 ₁ 2)	94.3.788 (P4' ₂ 2 ₁ 2')		D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	94.5.790 (P4' ₂ 2' ₁ 2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	94.4.789 (P4 ₂ 2' ₁ 2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#95 (P4 ₃ 22)	95.3.795 (P4' ₃ 22')		D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	95.5.797 (P4' ₃ 2'2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	95.4.796 (P4 ₃ 2'2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#96 (P4 ₃ 2 ₁ 2)	96.3.802 (P4' ₃ 2 ₁ 2')		D_2	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	96.5.804 (P4' ₃ 2' ₁ 2)		D_2	C_{4z}	2D d_{xy}^P -wave	
	96.4.803 (P4 ₃ 2' ₁ 2')		C_4	C_{2x}, C_{2y}	2D g^P -wave	
#97 (I422)	97.3.807 (I4'22')		D_2	C_{4z}	2D $d_{x^2-y^2}^I$ -wave	
	97.5.809 (I4'2'2)		D_2	C_{4z}	2D d_{xy}^I -wave	
	97.4.808 (I42'2')		C_4	C_{2x}, C_{2y}	2D g^I -wave	
#98 (I4 ₁ 22)	98.3.816 (I4' ₁ 22')		D_2	C_{4z}	2D $d_{x^2-y^2}^I$ -wave	
	98.5.818 (I4' ₁ 2'2)		D_2	C_{4z}	2D d_{xy}^I -wave	
	98.4.817 (I4 ₁ 2'2')		C_4	C_{2x}, C_{2y}	2D g^I -wave	

TABLE S9: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#99 (P4mm)	99.3.825 (P4'm'm)	C_{4v}	C_{2v}	C_{4z}	2D d_{xy}^P -wave	No
	99.4.826 (P4'mm')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	99.5.827 (P4m'm')		C_4	M_y, M_x	2D g^P -wave	
#100 (P4bm)	100.3.838 (P4'b'm)		C_{2v}	C_{4z}	2D d_{xy}^P -wave	
	100.4.839 (P4'bm')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	100.5.840 (P4b'm')		C_4	M_y, M_x	2D g^P -wave	
#101 (P4 ₂ cm)	101.3.847 (P4' ₂ c'm)		C_{2v}	C_{4z}	2D d_{xy}^P -wave	
	101.4.848 (P4' ₂ cm')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	101.5.849 (P4 ₂ c'm')		C_4	M_y, M_x	2D g^P -wave	
#102 (P4 ₂ nm)	102.3.854 (P4' ₂ n'm)		C_{2v}	C_{4z}	2D d_{xy}^P -wave	
	102.4.855 (P4' ₂ nm')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	102.5.856 (P4 ₂ n'm')		C_4	M_y, M_x	2D g^P -wave	
#103 (P4cc)	103.3.861 (P4'c'c)		C_{2v}	C_{4z}	2D d_{xy}^P -wave	
	103.4.862 (P4'cc')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	103.5.863 (P4c'c')		C_4	M_y, M_x	2D g^P -wave	
#104 (P4nc)	104.3.867 (P4'n'c)		C_{2v}	C_{4z}	2D d_{xy}^P -wave	
	104.4.868 (P4'nc')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	104.5.869 (P4n'c')		C_4	M_y, M_x	2D g^P -wave	
#105 (P4 ₂ mc)	105.3.873 (P4' ₂ m'c)		C_{2v}	C_{4z}	2D d_{xy}^P -wave	
	105.4.874 (P4' ₂ mc')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	105.5.875 (P4 ₂ m'c')		C_4	M_y, M_x	2D g^P -wave	
#106 (P4 ₂ bc)	106.3.880 (P4' ₂ b'c)		C_{2v}	C_{4z}	2D d_{xy}^P -wave	
	106.4.881 (P4' ₂ bc')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^P$ -wave	
	106.5.882 (P4 ₂ b'c')		C_4	M_y, M_x	2D g^P -wave	
#107 (I4mm)	107.3.885 (I4'm'm)		C_{2v}	C_{4z}	2D d_{xy}^I -wave	
	107.4.886 (I4'mm')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^I$ -wave	
	107.5.887 (I4m'm')		C_4	M_y, M_x	2D g^I -wave	
#108 (I4cm)	108.3.894 (I4'c'm)		C_{2v}	C_{4z}	2D d_{xy}^I -wave	
	108.4.895 (I4'cm')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^I$ -wave	
	108.5.896 (I4c'm')		C_4	M_y, M_x	2D g^I -wave	
#109 (I4 ₁ md)	109.3.903 (I4' ₁ m'd)		C_{2v}	C_{4z}	2D d_{xy}^I -wave	
	109.4.904 (I4' ₁ md')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^I$ -wave	
	109.5.905 (I4 ₁ m'd')		C_4	M_y, M_x	2D g^I -wave	
#110 (I4 ₁ cd)	110.3.908 (I4' ₁ c'd)		C_{2v}	C_{4z}	2D d_{xy}^I -wave	
	110.4.909 (I4' ₁ cd')		C_{2v}	C_{4z}	2D $d_{x^2-y^2}^I$ -wave	
	110.5.910 (I4 ₁ c'd')		C_4	M_y, M_x	2D g^I -wave	

TABLE S10: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#111 ($P\bar{4}2m$)	111.3.913 ($P\bar{4}'2'm$)	D_{2d}	C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	No
	111.4.914 ($P\bar{4}'2m'$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	111.5.915 ($P\bar{4}2'm'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#112 ($P\bar{4}2c$)	112.3.924 ($P\bar{4}'2'c$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	112.4.925 ($P\bar{4}'2c'$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	112.5.926 ($P\bar{4}2'c'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#113 ($P\bar{4}2_1m$)	113.3.931 ($P\bar{4}'2'_1m$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	113.4.932 ($P\bar{4}'2'_1m'$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	113.5.933 ($P\bar{4}2'_1m'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#114 ($P\bar{4}2_1c$)	114.3.938 ($P\bar{4}'2'_1c$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	114.4.939 ($P\bar{4}'2_1c'$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	114.5.940 ($P\bar{4}2'_1c'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#115 ($P\bar{4}m2$)	115.4.944 ($P\bar{4}'m2'$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	115.3.943 ($P\bar{4}'m'2$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	115.5.945 ($P\bar{4}m'2'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#116 ($P\bar{4}c2$)	116.4.954 ($P\bar{4}'c2'$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	116.3.953 ($P\bar{4}'c'2$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	116.5.955 ($P\bar{4}c'2'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#117 ($P\bar{4}b2$)	117.4.961 ($P\bar{4}'b2'$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	117.3.960 ($P\bar{4}'b'2$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	117.5.962 ($P\bar{4}b'2'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#118 ($P\bar{4}n2$)	118.4.968 ($P\bar{4}'n2'$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	118.3.967 ($P\bar{4}'n'2$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	118.5.969 ($P\bar{4}n'2'$)		S_4	C_{2x}, C_{2y}	2D g^P -wave	
#119 ($I\bar{4}m2$)	119.4.974 ($I\bar{4}'m2'$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^I -wave	
	119.3.973 ($I\bar{4}'m'2$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^I$ -wave	
	119.5.975 ($I\bar{4}m'2'$)		S_4	C_{2x}, C_{2y}	2D g^I -wave	
#120 ($I\bar{4}c2$)	120.4.981 ($I\bar{4}'c2'$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^I -wave	
	120.3.980 ($I\bar{4}'c'2$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^I$ -wave	
	120.5.982 ($I\bar{4}c'2'$)		S_4	C_{2x}, C_{2y}	2D g^I -wave	
#121 ($I\bar{4}2m$)	121.3.987 ($I\bar{4}'2'm$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^I -wave	
	121.4.988 ($I\bar{4}'2m'$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^I$ -wave	
	121.5.989 ($I\bar{4}2'm'$)		S_4	C_{2x}, C_{2y}	2D g^I -wave	
#122 ($I\bar{4}2d$)	122.3.996 ($I\bar{4}'2'd$)		C_{2v}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^I -wave	
	122.4.997 ($I\bar{4}'2d'$)		D_2	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^I$ -wave	
	122.5.998 ($I\bar{4}2'd'$)		S_4	C_{2x}, C_{2y}	2D g^I -wave	

TABLE S11: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#123 (P4/mmm)	123.4.1002 (P4'/mm'm)	D_{4h}	D_{2h}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	Yes
	123.5.1003 (P4'/mmm')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	123.7.1005 (P4/mm'm')		C_{4h}	M_y, M_x	2D g^P -wave	
#124 (P4/mcc)	124.4.1021 (P4'/mc'c)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d_{xy}^P -wave	
	124.5.1022 (P4'/mcc')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	124.7.1024 (P4/mc'c')		C_{4h}	M_y, M_x	2D g^P -wave	
#125 (P4/nbm)	125.4.1034 (P4'/nb'm)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	125.5.1035 (P4'/nbm')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	125.7.1037 (P4/nb'm')		C_{4h}	M_y, M_x	2D g^P -wave	
#126 (P4/nnc)	126.4.1047 (P4'/nn'c)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	126.5.1048 (P4'/nnc')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	126.7.1050 (P4'/nn'c')		C_{4h}	M_y, M_x	2D g^P -wave	
#127 (P4/mbm)	127.4.1056 (P4'/mb'm)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	127.5.1057 (P4'/mbm')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	127.7.1059 (P4'/mb'm')		C_{4h}	M_y, M_x	2D g^P -wave	
#128 (P4/mnc)	128.4.1069 (P4'/mn'c)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	128.5.1070 (P4'/mnc')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	128.7.1072 (P4'/mn'c')		C_{4h}	M_y, M_x	2D g^P -wave	
#129 (P4/nmm)	129.4.1078 (P4'/nm'm)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	129.5.1079 (P4'/nmm')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	129.7.1081 (P4'/nm'm')		C_{4h}	M_y, M_x	2D g^P -wave	
#130 (P4/ncc)	130.4.1091 (P4'/nc'c)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	130.5.1092 (P4'/ncc')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	130.7.1094 (P4'/nc'c')		C_{4h}	M_y, M_x	2D g^P -wave	
#131 (P4 ₂ /mmc)	131.4.1100 (P4' ₂ /mm'c)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	131.5.1101 (P4' ₂ /mmc')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	131.7.1103 (P4 ₂ /mm'c')		C_{4h}	M_y, M_x	2D g^P -wave	
#132 (P4 ₂ /mcm)	132.4.1113 (P4' ₂ /mc'm)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	132.5.1114 (P4' ₂ /mcm')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	132.7.1116 (P4 ₂ /mc'm')		C_{4h}	M_y, M_x	2D g^P -wave	
#133 (P4 ₂ /nbc)	133.4.1126 (P4' ₂ /nb'c)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	133.5.1127 (P4' ₂ /nbc')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	133.7.1129 (P4 ₂ /nb'c')		C_{4h}	M_y, M_x	2D g^P -wave	
#134 (P4 ₂ /nnm)	134.4.1135 (P4' ₂ /nn'm)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	134.5.1136 (P4' ₂ /nnm')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	134.7.1138 (P4 ₂ /nn'm')		C_{4h}	M_y, M_x	2D g^P -wave	
#135 (P4 ₂ /mbc)	135.4.1146 (P4' ₂ /mb'c)		D_{2h}	S_{4z}, S_{4z}^{-1}	2D d^P -wave	
	135.5.1147 (P4' ₂ /mbc')		D_{2h}	S_{4z}, S_{4z}^{-1}	2D $d_{x^2-y^2}^P$ -wave	
	135.7.1149 (P4 ₂ /mb'c')		C_{4h}	M_y, M_x	2D g^P -wave	

TABLE S12: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#136 (P4 ₂ /mnm)	136.4.1155 (P4 ₂ /mn'm)	<i>D</i> _{4h}	<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>P</i>} -wave	Yes
	136.5.1156 (P4 ₂ /mnm')		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>P</i>} _{<i>x</i>²−<i>y</i>²} -wave	
	136.7.1158 (P4 ₂ /mn'm')		<i>C</i> _{4h}	<i>M</i> _{<i>y</i>} , <i>M</i> _{<i>x</i>}	2D <i>g</i> ^{<i>P</i>} -wave	
#137 (P4 ₂ /nmc)	137.4.1164 (P4' ₂ /nm'c)		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>P</i>} -wave	
	137.5.1165 (P4' ₂ /nmc')		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>P</i>} _{<i>x</i>²−<i>y</i>²} -wave	
	137.7.1167 (P4 ₂ /nm'c')		<i>C</i> _{4h}	<i>M</i> _{<i>y</i>} , <i>M</i> _{<i>x</i>}	2D <i>g</i> ^{<i>P</i>} -wave	
#138 (P4 ₂ /ncm)	138.4.1173 (P4' ₂ /nc'm)		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>P</i>} -wave	
	138.5.1174 (P4' ₂ /ncm')		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>P</i>} _{<i>x</i>²−<i>y</i>²} -wave	
	138.7.1176 (P4 ₂ /nc'm')		<i>C</i> _{4h}	<i>M</i> _{<i>y</i>} , <i>M</i> _{<i>x</i>}	2D <i>g</i> ^{<i>P</i>} -wave	
#139 (I4/mmm)	139.4.1182 (I4'/mm'm)		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} -wave	
	139.5.1183 (I4'/mmm')		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} _{<i>x</i>²−<i>y</i>²} -wave	
	139.7.1185 (I4/mm'm')		<i>C</i> _{4h}	<i>M</i> _{<i>y</i>} , <i>M</i> _{<i>x</i>}	2D <i>g</i> ^{<i>I</i>} -wave	
#140 (I4/mcm)	140.4.1199 (I4'/mc'm)		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} -wave	
	140.5.1200 (I4'/mcm')		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} _{<i>x</i>²−<i>y</i>²} -wave	
	140.7.1202 (I4/mc'm')		<i>C</i> _{4h}	<i>M</i> _{<i>y</i>} , <i>M</i> _{<i>x</i>}	2D <i>g</i> ^{<i>I</i>} -wave	
#141 (I4 ₁ /amd)	141.4.1216 (I4' ₁ /am'd)		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} -wave	
	141.5.1217 (I4' ₁ /amd')		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} _{<i>x</i>²−<i>y</i>²} -wave	
	141.7.1219 (I4 ₁ /am'd')		<i>C</i> _{4h}	<i>M</i> _{<i>y</i>} , <i>M</i> _{<i>x</i>}	2D <i>g</i> ^{<i>I</i>} -wave	
#142 (I4 ₁ /acd)	142.4.1225 (I4' ₁ /ac'd)		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} -wave	
	142.5.1226 (I4' ₁ /acd')		<i>D</i> _{2h}	<i>S</i> _{4z} , <i>S</i> _{4z} ^{−1}	2D <i>d</i> ^{<i>I</i>} _{<i>x</i>²−<i>y</i>²} -wave	
	142.7.1228 (I4 ₁ /ac'd')		<i>C</i> _{4h}	<i>M</i> _{<i>y</i>} , <i>M</i> _{<i>x</i>}	2D <i>g</i> ^{<i>I</i>} -wave	
#143 (P3)	×	<i>C</i> ₃	×	×	×	No
#144 (P3 ₁)						
#145 (P3 ₂)						
#146 (R3)						
#147 (P3̄)	×	<i>S</i> ₆	×	×	×	Yes
#148 (R3̄)						
#149 (P312)	149.3.1253 (P312')	<i>D</i> ₃	<i>C</i> ₃	<i>C</i> ₂ '	3D <i>g</i> ^{<i>P</i>} _{<i>M</i>} -wave	No
#150 (P321)	150.3.1257 (P32'1)				3D <i>g</i> ^{<i>P</i>} _{<i>K</i>} -wave	
#151 (P3 ₁ 12)	151.3.1261 (P3 ₁ 12')				3D <i>g</i> ^{<i>P</i>} _{<i>M</i>} -wave	
#152 (P3 ₁ 21)	152.3.1265 (P3 ₁ 2'1)				3D <i>g</i> ^{<i>P</i>} _{<i>K</i>} -wave	
#153 (P3 ₂ 12)	153.3.1269 (P3 ₂ 12')				3D <i>g</i> ^{<i>P</i>} _{<i>M</i>} -wave	
#154 (P3 ₂ 21)	154.3.1273 (P3 ₂ 2'1)				3D <i>g</i> ^{<i>P</i>} _{<i>K</i>} -wave	
#155 (R32)	155.3.1277 (R32')				3D <i>g</i> ^{<i>R</i>} -wave	
#156 (P3m1)	156.3.1281 (P3m'1)	<i>C</i> _{3v}	<i>C</i> ₃	<i>M</i> '	3D <i>g</i> ^{<i>P</i>} _{<i>M</i>} -wave	No
#157 (P31m)	157.3.1286 (P31m')				3D <i>g</i> ^{<i>P</i>} _{<i>K</i>} -wave	
#158 (P3c1)	158.3.1291 (P3c'1)				3D <i>g</i> ^{<i>P</i>} _{<i>M</i>} -wave	
#159 (P31c)	159.3.1294 (P31c')				3D <i>g</i> ^{<i>P</i>} _{<i>K</i>} -wave	
#160 (R3m)	160.3.1297 (R3m')				3D <i>g</i> ^{<i>R</i>} -wave	
#161 (R3c)	161.3.1302 (R3c')					

TABLE S13: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#162 (P $\bar{3}$ 1m)	162.5.1307 (P $\bar{3}$ 1m')	D_{3d}	S_6	C_2', M'	3D g_K^P -wave	Yes
#163 (P $\bar{3}$ 1c)	163.5.1314 (P $\bar{3}$ 1c')				3D g_K^P -wave	
#164 (P $\bar{3}$ m1)	164.5.1319 (P $\bar{3}$ m'1)				3D g_M^P -wave	
#165 (P $\bar{3}$ c1)	165.5.1326 (P $\bar{3}$ c'1)				3D g_M^P -wave	
#166 (R $\bar{3}$ m)	166.5.1331 (R $\bar{3}$ m')				3D g^R -wave	
#167 (R $\bar{3}$ c)	167.5.1338 (R $\bar{3}$ c')					
#168 (P6)	168.3.1341 (P6')	C_6	C_3	C_{6z}	3D g^P -wave	No
#169 (P6 ₁)	169.3.1346 (P6' ₁)					
#170 (P6 ₅)	170.3.1349 (P6' ₅)					
#171 (P6 ₂)	171.3.1352 (P6' ₂)					
#172 (P6 ₄)	172.3.1357 (P6' ₄)					
#173 (P6 ₃)	173.3.1362 (P6' ₃)					
#174 (P $\bar{6}$)	174.3.1365 (P $\bar{6}'$)	C_{3h}	C_3	M_z	3D g^P -wave	No
#175 (P6/m)	175.5.1371 (P6'/m')	C_{6h}	S_6	C_{6z}	3D g^P -wave	Yes
#176 (P6 ₃ /m)	176.5.1378 (P6' ₃ /m')					
#177 (P6 ₂₂)	177.3.1381 (P6'2'2)	D_6	D_3	C_{6z}	3D g_K^P -wave	No
	177.4.1382 (P6'22')		D_3	C_{6z}	3D g_M^P -wave	
	177.5.1383 (P6 ₂ '2')		C_6	C_2', C_2''	2D i^P -wave	
#178 (P6 ₁ 22)	178.3.1388 (P6' ₁ 2'2)		D_3	C_{6z}	3D g_K^P -wave	
	178.4.1389 (P6' ₁ 22')		D_3	C_{6z}	3D g_M^P -wave	
	178.5.1390 (P6 ₁ 2'2')		C_6	C_2', C_2''	2D i^P -wave	
#179 (P6 ₅ 22)	179.3.1393 (P6' ₅ 2'2)		D_3	C_{6z}	3D g_K^P -wave	
	179.4.1394 (P6' ₅ 22')		D_3	C_{6z}	3D g_M^P -wave	
	179.5.1395 (P6 ₅ 2'2')		C_6	C_2', C_2''	2D i^P -wave	
#180 (P6 ₂ 22)	180.3.1398 (P6' ₂ 2'2)		D_3	C_{6z}	3D g_K^P -wave	
	180.4.1399 (P6' ₂ 22')		D_3	C_{6z}	3D g_M^P -wave	
	180.5.1400 (P6 ₂ 2'2')		C_6	C_2', C_2''	2D i^P -wave	
#181 (P6 ₄ 22)	181.3.1405 (P6' ₄ 2'2)		D_3	C_{6z}	3D g_K^P -wave	
	181.4.1406 (P6' ₄ 22')		D_3	C_{6z}	3D g_M^P -wave	
	181.5.1407 (P6 ₄ 2'2')		C_6	C_2', C_2''	2D i^P -wave	
#182 (P6 ₃ 22)	182.3.1412 (P6' ₃ 2'2)		D_3	C_{6z}	3D g_K^P -wave	
	182.4.1413 (P6' ₃ 22')		D_3	C_{6z}	3D g_M^P -wave	
	182.5.1414 (P6 ₃ 2'2')		C_6	C_2', C_2''	2D i^P -wave	

TABLE S14: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#183 (P6mm)	183.3.1417 (P6'm'm)	C_{6v}	C_{3v}	C_{6z}	3D g_K^P -wave	No
	183.4.1418 (P6'mm')		C_{3v}	C_{6z}	3D g_M^P -wave	
	183.5.1419 (P6m'm')		C_6	M_v, M_d	2D i^P -wave	
#184 (P6cc)	184.3.1426 (P6'c'c)		C_{3v}	C_{6z}	3D g_K^P -wave	
	184.4.1427 (P6'cc')		C_{3v}	C_{6z}	3D g_M^P -wave	
	184.5.1428 (P6c'c')		C_6	M_v, M_d	2D i^P -wave	
#185 (P6 ₃ cm)	185.3.1431 (P6' ₃ c'm)		C_{3v}	C_{6z}	3D g_K^P -wave	
	185.4.1432 (P6' ₃ cm')		C_{3v}	C_{6z}	3D g_M^P -wave	
	185.5.1433 (P6 ₃ c'm')		C_6	M_v, M_d	2D i^P -wave	
#186 (P6 ₃ mc)	186.3.1436 (P6' ₃ m'c)		C_{3v}	C_{6z}	3D g_K^P -wave	
	186.4.1437 (P6' ₃ mc')		C_{3v}	C_{6z}	3D g_M^P -wave	
	186.5.1438 (P6 ₃ m'c')		C_6	M_v, M_d	2D i^P -wave	
#187 (P6m2)	187.3.1441 (P6'm'2)	D_{3h}	D_3	M_z	3D g_K^P -wave	No
	187.4.1442 (P6'm2')		D_{3v}	M_z	3D g_M^P -wave	
	187.5.1443 (P6m'2')		C_{3h}	M_v	2D i^P -wave	
#188 (P6c2)	188.3.1448 (P6'c'2)		D_3	M_z	3D g_K^P -wave	
	188.4.1449 (P6'c2')		D_{3v}	M_z	3D g_M^P -wave	
	188.5.1450 (P6c'2')		C_{3h}	M_v	2D i^P -wave	
#189 (P62m)	189.3.1453 (P6'2'm)		D_{3v}	M_z	3D g_K^P -wave	
	189.4.1454 (P6'2m')		D_3	M_z	3D g_M^P -wave	
	189.5.1455 (P62'm')		C_{3h}	M_v	2D i^P -wave	
#190 (P62c)	190.3.1460 (P6'2'c)		D_{3v}	M_z	3D g_K^P -wave	
	190.4.1461 (P6'2c')		D_3	M_z	3D g_M^P -wave	
	190.5.1462 (P62'c')		C_{3h}	M_v	2D i^P -wave	
#191 (P6/mmm)	191.6.1468 (P6'/m'm'm)	D_{6h}	D_{3d}	C_{6z}	3D g_K^P -wave	Yes
	191.7.1469 (P6'/m'mm')		D_{3d}	C_{6z}	3D g_M^P -wave	
	191.8.1470 (P6/mm'm')		C_{6h}	M_v	2D i^P -wave	
#192 (P6/mcc)	192.6.1481 (P6'/m'c'c)		D_{3d}	C_{6z}	3D g_K^P -wave	
	192.7.1482 (P6'/m'cc')		D_{3d}	C_{6z}	3D g_M^P -wave	
	192.8.1483 (P6/mc'c')		C_{6h}	M_v	2D i^P -wave	
#193 (P6 ₃ /mcm)	193.6.1490 (P6' ₃ /m'c'm)		D_{3d}	C_{6z}	3D g_K^P -wave	
	193.7.1491 (P6' ₃ /m'cm')		D_{3d}	C_{6z}	3D g_M^P -wave	
	193.8.1492 (P6 ₃ /mc'm')		C_{6h}	M_v	2D i^P -wave	
#194 (P6 ₃ /mmc)	193.6.1499 (P6' ₃ /m'm'c)		D_{3d}	C_{6z}	3D g_K^P -wave	
	193.7.1500 (P6' ₃ /m'mc')		D_{3d}	C_{6z}	3D g_M^P -wave	
	193.8.1501 (P6 ₃ /mm'c')		C_{6h}	M_v	2D i^P -wave	

TABLE S15: Classification of altermagnetic space groups and their spin-splitting characteristics.

Space group	ASG	Point group	Subgroup	Coset	Type	Inversion symmetry
#195 (P23)	$\times$	T	$\times$	$\times$	$\times$	No
#196 (F23)						
#197 (I23)						
#198 (P2 ₁ 3)						
#199 (I2 ₁ 3)						
#200 (Pm $\bar{3}$)	$\times$	T_h	$\times$	$\times$	$\times$	Yes
#201 (Pn $\bar{3}$)						
#202 (Fm $\bar{3}$)						
#203 (Fd $\bar{3}$)						
#204 (Im $\bar{3}$)						
#205 (Pa $\bar{3}$)						
#206 (Ia $\bar{3}$)						
#207 (P432)	207.3.1544 (P4'32')	O	T	C₄	3D i^P -wave	No
#208 (P4 ₂ 32)	208.3.1548 (P4' ₂ 32')				3D i^F -wave	
#209 (F432)	209.3.1552 (F4'32')				3D i^I -wave	
#210 (F4 ₁ 32)	210.3.1555 (F4' ₁ 32')				3D i^P -wave	
#211 (I432)	211.3.1558 (I4'32')				3D i^I -wave	
#212 (P4 ₃ 32)	212.3.1563 (P4' ₃ 32')					
#213 (P4 ₁ 32)	213.3.1566 (P4' ₁ 32')					
#214 (I4 ₁ 32)	214.3.1569 (I4' ₁ 32')					
#215 (P $\bar{4}$ 3m)	215.3.1574 (P $\bar{4}$ '3m')	T_d	T	S₄, M_d	3D i^P -wave	No
#216 (F $\bar{4}$ 3m)	216.3.1579 (F $\bar{4}$ '3m')				3D i^F -wave	
#217 (I $\bar{4}$ 3m)	217.3.1582 (I $\bar{4}$ '3m')				3D i^I -wave	
#218 (P $\bar{4}$ 3n)	218.3.1587 (P $\bar{4}$ '3n')				3D i^P -wave	
#219 (F $\bar{4}$ 3c)	219.3.1590 (F $\bar{4}$ '3c')				3D i^F -wave	
#220 (I $\bar{4}$ 3d)	220.3.1593 (I $\bar{4}$ '3d')				3D i^I -wave	
#221 (Pm $\bar{3}$ m)	221.4.1597 (Pm $\bar{3}$ m')	O_h	T_h	C₄, M_d	3D i^P -wave	Yes
#222 (Pn $\bar{3}$ n)	222.4.1604 (Pn $\bar{3}$ n')					
#223 (Pm $\bar{3}$ n)	223.4.1609 (Pm $\bar{3}$ n')					
#224 (Pn $\bar{3}$ m)	224.4.1614 (Pn $\bar{3}$ m')					
#225 (Fm $\bar{3}$ m)	225.4.1621 (Fm $\bar{3}$ m')					
#226 (Fm $\bar{3}$ c)	226.4.1626 (Fm $\bar{3}$ c')				3D i^F -wave	
#227 (Fd $\bar{3}$ m)	227.4.1631 (Fd $\bar{3}$ m')					
#228 (Fd $\bar{3}$ c)	228.4.1636 (Fd $\bar{3}$ c')					
#229 (Im $\bar{3}$ m)	229.4.1641 (Im $\bar{3}$ m')					
#230 (Ia $\bar{3}$ d)	230.4.1650 (Ia $\bar{3}$ d')				3D i^I -wave	

TABLE S16: Classification of altermagnetic space groups and their spin-splitting characteristics.