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Carnapian Neo-Fregeanism and the Bad Company Objection

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Hume's Principle, which is the principle on which neo-Fregeans wish to build arithmetic, is an abstraction principle. Many abstraction principles are unacceptable to the neo-Fregean, for instance because they are inconsistent or because they are inconsistent together with Hume's Principle. What differentiates Hume's Principle from these unacceptable abstraction principles? This question, which captures the so-called bad company objection, has proved difficult to answer and continues to plague the neo-Fregean programme. In this paper, I draw on the philosophy of Rudolf Carnap to develop a Carnapian neo-Fregeanism. I show how the Carnapian neo-Fregean can deal relatively straightforward with the bad company objection. I also consider why a Carnapian should be attracted to the neo-Fregean programme and, finally, argue that some of the pressing problems which have developed from the bad company objection are no issue for the Carnapian neo-Fregean.

1 Introduction

The axioms of second-order Peano Arithmetic can be derived, within an appropriate second-order logic, from Hume's Principe (HP):[1]

$$\forall F \forall G(NxFx = NxGx \leftrightarrow F \approx G)$$

[1] See for instance Wright (1983: 4.xix), Boolos (1990) and Heck (2011) for details.

Here 'Nx_x' intuitively stands for 'the number of ___' where what is to fill the blank stands for a (first-order) concept, 'F' and 'G' are variables for such concepts, and '___ ≈ …' stands for '___ is equinumerous with …' where what fills the blanks will again stand for (first-order) concepts. So HP says that the number of Fs is the number of Gs if and only if the Fs and the Gs are equinumerous. Within a logic that allows quantification over two-place relations, equinumerosity can be given a definition in purely logical terms: there is a bijection between (the extensions of) the two concepts.[2]

As part of his logicist project of reducing arithmetic to logic, Frege was the first to indicate how (without formally proving that) the axioms of Peano Arithmetic could be derived from HP with appropriate definitions ([1884] 1953: §§70–83). But he also argued that HP, although true, could not by itself be the foundation on which to build arithmetic. His argument ([1884] 1953: §56), now known as the Julius Caesar objection, has received considerable attention (see, for instance, Heck 1997b; Studd 2023; and the references in these). In its shortest form, it is this: HP does not entail that the number 1 is different from Julius Caesar, even though they plainly are different. Hence HP does not settle all that we should want settled about the nature of the numbers. In response to this argument, Frege gave an *explicit* definition of 'Nx_x' that was intended to settle that Julius Caesar is not any number. Unfortunately, his explicit definition relied on an inconsistent background framework of extensions (or slightly more precisely, value-ranges of functions; *Wertverläufe* in German).

Neo-Fregeans, most prominently Bob Hale and Crispin Wright, have attempted to defend a version of Frege's logicism by rejecting the need to find foundations firmer than HP. They argue that HP, as an implicit definition of the term-forming operator 'Nx_x' on which arithmetic can be built with further appropriate explicit definitions, yields all the logicist should want, so that Frege's inconsistent framework of extensions can be avoided (Wright 1983; Hale and Wright 2001; MacBride 2003). Unsurprisingly, the neo-Fregean programme prompted a significant literature on the status of HP: can it stand on its own as an implicit definition without creating serious problems for the logicism that is intended to be erected on it?

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[2] Formally: $\exists R(\forall x(Fx \rightarrow \exists y(Gy \wedge xRy)) \wedge \forall x(Fx \rightarrow \neg\exists y\exists z(y \neq z \wedge Gy \wedge Gz \wedge xRy \wedge xRz)) \wedge \forall x(Gx \rightarrow \exists y(Fy \wedge yRx)) \wedge \forall x\forall y((x \neq y \wedge Fx \wedge Fy) \rightarrow \neg\exists z(Gz \wedge xRz \wedge yRz)))$.

One prominent and relatively straightforward argument against the acceptability of HP as a foundation for arithmetical logicism is known as the *bad company objection.*[3] This objection starts with the observation that HP is an abstraction principle. Abstraction principles introduce a term-forming operator via an equivalence relation. In HP, the term-forming operator that is introduced is 'the number of ___' ('Nx_x') and the equivalence relation is '___ is equinumerous with …' ('__ $\approx$ …'). Frege himself also considered another abstraction principle ([1884] 1953: §§64–68): one may introduce 'the direction of ___', where what fills the blank stands for a line segment in a plane, by saying that the direction of AB is the direction of CD iff AB and CD are parallel. In symbols this may become (the universal closure of):

$$d(AB) = d(CD) \leftrightarrow AB \parallel CD$$

Here '___ $\parallel$ …' stands for '___ is parallel to …', which is an equivalence relation on line segments. It is well-known that there are also abstraction principles which lead to contradiction. Here too we can use an example from Frege's work, namely his Basic Law V, which introduces value-ranges:

$$\forall f \forall g (\dot{\varepsilon} f\varepsilon = \dot{\varepsilon} g\varepsilon \leftrightarrow \forall x (fx = gx))$$

Here '$f\_$' and '$g\_$' are variables for (unary first-order) functions and '$\dot{\varepsilon}\_\varepsilon$' stands for 'the value-range of __' where what fills the blank stands for a (unary first-order) function. The equivalence relation is the second-order relation expressed by '$\forall x(\_x = ..x)$' where what fills the blanks stand again for (unary first-order) functions. Russell famously showed, in a letter to Frege, that Basic Law V is inconsistent.

In its simplest form, the bad company objection simply challenges the neo-Fregean to distinguish in a principled manner between HP and Basic Law V. HP is not supposed to be an empirical truth, but an analytic or conceptual truth. But it shares its logical form with inconsistent abstraction principles such as Basic Law V, so HP cannot be true in virtue of its logical form. What, then, distinguishes HP from Basic Law V? Why is HP acceptable and Basic Law V not?

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[3] The literature on this topic is large. For two brief but helpful introductions, see Linnebo (2009) and Hale and Wright (2001: 16–20); a more elaborate introduction is MacBride (2003: §8). For some early statements of the bad company objection, see Boolos (1990: 273; 1997), Dummett (1991: 188–89; 1998) and Field (1989: 158).

A simple answer naturally suggests itself: HP is consistent, Basic Law V is not. Consistent abstraction principles are acceptable, inconsistent ones should be rejected. This simple answer might have satisfied, had it not been shown that there are singly consistent abstraction principles—for example, the *parity principle* (Boolos 1990: 273) and the *nuisance principle* (Wright 1997)—each of which is jointly inconsistent with HP.[4] The details of these principles do not matter here; what does matter is that they appear to prevent consistency from being the distinguishing mark of acceptable abstraction principles. We cannot prefer HP over the parity or nuisance principle on the grounds that only HP is consistent, and to disqualify the parity and nuisance principles because each of them is jointly inconsistent with HP is only to beg the question. The bad company objection thus remains: how do we determine which abstraction principles are acceptable?

A substantial literature discusses technical criteria that might allow neo-Fregeans to distinguish in a principled manner between HP and its bad company such as the parity and nuisance principles. Philosophy of this genre ranges from the technical to the *technical*. There are Wright's original suggestions that the acceptable abstraction principles should be *conservative* (1997) or *modest* (1999), Alan Weir's suggestion they should be *irenic* (2003), Roy Cook's suggestion that the acceptable principles are *strongly stable* and thus *maximally strictly symmetrically class conservative* (2012; 2016), Hannes Leitgeb's attempt to sidestep the acceptable/unacceptable dichotomy through *groundedness* (2016), and so on.[5] My concern in this paper is not with such technical attempts and so I will not put flesh on these italics, the admirable formal artistry involved notwithstanding.

Instead, I will argue that there is an appealing perspective from which there is little need for such technical criteria. The perspective I mean is inspired by the philosophy of Rudolf Carnap and decidedly pragmatic. My goal is not to defend Carnapianism *per se*, but, much more modestly, to argue that it enables an attractive pragmatic solution to the bad company objection against neo-Fregeanism. Those who already like the Carnapian outlook may thus find in this paper yet another philosophical issue that it resolves rather elegantly; those who are

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[4] Both principles are satisfiable only in a finite model, whereas HP demands an infinite domain. Where it matters, I will use 'inconsistent' to mean unsatisfiable. The objection based on *parity* and *nuisance* is also sometimes called the 'embarrassment of riches objection', e.g. in Weir (2003). See also Heck (1992).

[5] Some of these do not address consistency directly, but instead centre on, for instance, domain sizes or expansions of the language.

already invested in neo-Fregeanism may find in this paper reasons to take the Carnapian attitude seriously.

I explain the basics of the Carnapian perspective and the compelling light I think it shines on the bad company objection in §2 and §3, respectively. Central to these is the Carnapian's pragmatic attitude: abstraction principles are to be assessed on their usefulness. In §4, I then discuss what, for the Carnapian, the pragmatic value of the neo-Fregean programme consists in: why is HP, from the Carnapian perspective, a useful abstraction principle? As part of this discussion, I offer an interpretation of Carnap's suggestion that epistemological clarity is gained when arithmetic is rendered analytic, and briefly discuss the epistemological side of the bad company objection raised by Philip Ebert and Stewart Shapiro (2009). In §5, I introduce two further objections against neo-Fregeanism: what has been called the 'good company objection' (Mancosu 2015) and the proliferation problem (Hawley 2007; Heck 2017). I show how the Carnapian neo-Fregean overcomes these further challenges and emphasise the Carnapian's relaxed attitude towards abstract entities. In the concluding §6, I briefly explain how my Carnapian neo-Fregeanism relates to three similar positions that have been considered in the literature and consider what it is that makes the Carnapian approach unique.

2 Carnapianism: the Basics

To explain the Carnapian perspective on neo-Fregeanism and the bad company objection, I will briefly introduce two elements of Carnap's mature philosophy. The first important element is the notion of a *linguistic framework*, which Carnap develops in his landmark paper 'Empiricism, semantics, and ontology' (1950). A linguistic framework consists of primitive expressions which are governed by suitable grammatical rules, and semantic rules of assessment for the declarative sentences that can be created with those primitive expressions and grammatical rules. A linguistic framework may for instance contain statements about the physical world by linking, through the rules of assessment, certain basic sentences to particular observations. Equally, a framework may be wholly abstract and its truths be determined solely by its axioms—sentences simply declared true—and logic.

Relative to a framework—if it is set up well—a string of symbols which is recognised as a sentence by the framework will be associated with a relatively clear notion of evidence, which means that, for any such sentence we may wish to query, there is, again relative to the framework, 'agreement as to the investigatory results—the experimental outcomes or computations, for example—that would resolve it' (Ricketts 1982: 118). To determine whether the sentence is true (relative to the framework) might of course be complicated or even impossible for any one human being. But the framework indicates at least how it could in principle be determined. In this way, linguistic frameworks are supposed to be different from natural languages, which contain many sentences which are vague ('a collection of 200 contiguous sand grains is a heap') or go without any notion of evidence ('Judith is a prime number').

The second crucial part of the Carnapian perspective is the rejection of an absolute criterion by which to judge frameworks and the pragmatic attitude that follows from this rejection. There is no framework-independent standpoint from which one can judge whether a framework is correct or incorrect, true or false, 'gets it right' or not. After all, the moment someone gives some theoretical criteria by which a framework is to be judged, the judgment will essentially happen *within* a framework: the theoretical criteria given simply specify the semantic rules of assessment of a latent background framework which introduces this notion of correctness. The question of correctness would then reapply to this framework and clearly it is possible to create many different 'background' frameworks—frameworks which introduce a notion of correctness for frameworks—which say of themselves that they are the correct framework to judge correctness of frameworks. Hence the choice between frameworks is not principally informed by theoretical matters; instead, frameworks are to be judged on their utility relative to a goal. Broadly theoretical matters—e.g. how simple and expressive is the framework?—may of course inform that pragmatic judgment, but they are not taken to have absolute force.

The Carnapian's pragmatism is intertwined with a critical tolerance. The Carnapian is tolerant because the rejection of a framework-independent perspective means that you are free to work with whatever framework you please in the sense that your choice cannot be an alethic or epistemological mistake.[6] There is no such thing as choosing the wrong framework, unless 'wrong' is taken to mean: totally useless given your goal. Yet this tolerance is critical exactly because of its pragmatic emphasis. The goals we choose to pursue, and the frameworks we use to pursue them, are always open to criticism. For example, years

[6] See besides 'Empiricism, semantics, and ontology' (1950) also Carnap's principle of tolerance in *Logical Syntax of Language* ([1937] 2000: §17)

spent developing a metaphysical framework which 'captures the fundamental structure of reality' but has no real utility whatsoever is, to the Carnapian, years wasted.[7] Such endeavours can be criticised from a pragmatic point of view: in this way, the Carnapian spirit is still anti-metaphysical.[8]

An example may help to illustrate. Philosophers have been much concerned with propositions. Do they exist? If so, what are they? Are they structured or unstructured? From the Carnapian perspective, these questions should be asked relative to a linguistic framework because asked as questions of English, it is unclear how to settle them. In other words, such questions can only be answered interestingly when the phrases 'to exist', 'proposition', 'structured', etc. are made more precise than they are in natural language. Of course we are free to set up the rules of a Carnapian framework for propositions however we want. We could set up the rules such that 'this is a proposition' is true whenever 'this' refers, in context, to a building that contains at least ten chairs. This framework would plainly be rather uninteresting because it severs completely the connection with the English use of 'proposition'. Better, then, to have more reasonable rules, for instance one which states something like:

> If S is a declarative sentence of English, then S expresses a proposition.

With such a rule, and the identification of existence with the existential quantifier, it is easy to determine that, within the proposed framework, propositions exist. Another rule might tell us when two declarative sentences of English express the same proposition, perhaps with some abstraction principle (quantifying over sentences) of the form

$$\forall S \forall T(prop(S) = prop(T) \leftrightarrow \_\_)$$

And if we want the framework also to settle whether propositions are structured or not, we simply need axioms that determine this or provide sentences of the form 'x is a structured proposition' with a clear notion of evidence. Crucially, there is no such thing as getting it right. If my framework says that propositions are structured, and yours that they are

[7] It may even be harmful because it suggests a finality which is at odds with the changing nature of scientific and political progress.
[8] See, for instance, Cohen and Marschall (2023) and Morris (2018).

not, there is no interesting question which of us got it right—which framework we should prefer depends only on the work we hope to do with it.

3 How Carnapians Show Bad Company the Door

My brief sketch of the Carnapian attitude is already sufficient to showcase the Carnapian way out of the bad company objection. The Carnapian rejects the idea that 'is HP true?' is a good question when asked independently of a linguistic framework—how on earth is it to be settled?[9]—and dismisses the question when asked relative to such a framework as uninteresting because trivial—typically, HP is either an axiom or straightforwardly follows from one, and in either case the answer is simply 'yes'. Much the same holds for the questions 'is the nuisance principle true?' or 'is the parity principle true?'. The only interesting questions that remain concern the tenability of frameworks which incorporate such abstraction principles. The question which abstraction principles should be accepted thus becomes, for the Carnapian, the question which frameworks should be accepted. Since the acceptance of linguistic frameworks is a pragmatic affair, the Carnapian focuses, without a doubt to everyone's great surprise, on the pragmatic value of the relevant frameworks.

What are the relevant frameworks? Let us consider, for the moment, these four: (i) the neo-Fregean's arithmetical framework which has HP as its only non-logical axiom, (ii) Frege's framework which has Basic Law V as an axiom, (iii) the parity framework which has as its only non-logical axiom Boolos' parity principle, and (iv) the nuisance framework which has *both* HP and the nuisance principle among its axioms. Which of these frameworks should be accepted?

Because this last question is, for the Carnapian, a pragmatic question, it only makes sense to ask it relative to a purpose. Which of the frameworks should we accept *given that we want to do so-and-so*? This point is not at all trivial. Why are philosophers interested in neo-Fregeanism? What do they want to do? What would a successful neo-Fregeanism show? In fact, Carnap had answers to such questions, which I explain in §4. For the moment, we can set them aside because it is not necessary to answer them to develop the Carnapian solution to the bad

[9] Of course there have been attempts to get a grasp of the cognitive project on which the neo-Fregean has embarked. See, for instance, Pedersen (2016).

company objection. We will simply assume that some goal has been formulated.

From the Carnapian perspective, the bad company objection is almost entirely uninteresting. A principled reason to reject Frege's framework (ii), for instance, is immediately available: frameworks that are inconsistent tend to be rather useless—assuming a classical logic, every sentence comes out as true, making it unworkable—and Frege's framework (ii) is inconsistent.[10] The parity and nuisance principles are only marginally more interesting. The nuisance framework (iv), which incorporates both HP and the nuisance principle, is again inconsistent and therefore not very interesting. The parity framework (iii), which adds only the parity principle to the background logic, does not suffer from inconsistency-induced uselessness. If this framework can be shown to have some practical or even purely mathematical interest, then let us work with it; if not, then not. Exactly the same holds for the neo-Fregean's arithmetical framework (i) which adds only HP to the background logic: its acceptability depends on nothing but its expediency. Because the Carnapian confronts the abstraction principles only in the context of a framework presented as a tool, the joint inconsistency of HP and the parity and nuisance principles is only a problem if they are all part of a single framework.

(In fact, inconsistent frameworks can be useful. Naïve set theory can for example be useful to get beginners to appreciate the absolute basics of axiomatic set theory through a familiarity with a naïve concept of set. From the Carnapian perspective, if you set yourself this purely pedagogical goal, you need not hesitate to use naïve set theory—this is not a crime against the absolute, objective metaphysical structure of mathematical reality because there is no such thing![11] Instead, the situation is analogous to that of teaching someone how to drive a car by letting them practice in a simulation: becoming familiar with the simulation is useful insofar as it facilitates learning to drive a car, which is the far more important skill. Of course such inconsistent frameworks

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[10] There is an alternative Carnapian perspective, which is to deny any putative framework with inconsistent axioms the label 'linguistic framework'. Whether we admit inconsistent frameworks or not is, ultimately, a pragmatic decision about the various ways of making 'linguistic framework' precise.

[11] The story must be slightly more complicated: 'the absolute objective metaphysical structure of mathematical reality' may itself be part of a framework and, within that framework, all sorts of things may be said truly with that phrase (that there is this kind of a structure, for example). The question then becomes what the use of such a metaphysical framework would be. See Cohen and Marschall (2023) for critical discussion of such attempts to turn nothing to something.

do not facilitate rational debate, but that is, in the imagined scenarios, not what they are being used for.)

The Carnapian emphasis on utility highlights an interesting question: why should the neo-Fregean arithmetical framework, with the numbers introduced as abstracted from an equivalence relation over concepts in the way indicated by HP, be useful or interesting? As I mentioned in the previous section, Carnap himself had an answer to this question, and we will turn to that answer in the next section. It is worth stressing now, however, that, from the Carnapian perspective, it would be a mistake to argue in favour of the neo-Fregean framework (i) on the grounds that this framework is correct, or 'gets it right', in some absolute sense. There just is not anything to 'get right' in any absolute sense. Of course one could develop some precise procedure for deciding whether the neo-Fregean framework is correct—perhaps by polling mathematicians and philosophers—but this would amount to nothing more than making the notion of 'correct' tractable by internalising it to a new framework, namely one for correctness. The question would then once again be: why should this framework, which gives a precise meaning to 'correct', be useful or interesting? This is the Carnapian's critical tolerance in action. But crucially, the Carnapian neo-Fregean can draw on Carnap's philosophy to argue directly that frameworks in which HP is the basic axiom from which arithmetic is derived are importantly interesting and useful.

4 The Value of Hume's Principle and Logicism

For Carnap, the value of HP lies in its resolution of one of the central conundrums in the philosophy of mathematics: if numbers are abstract objects, how on earth can they be so useful in our dealings with the concrete physical world? This vexed question is naturally raised for a scientific framework which, besides physical concepts, incorporates arithmetic immediately via the Peano Axioms: the axioms do not reveal how the arithmetical truths derivable from them relate to the physical world. But when arithmetic is introduced through Hume's Principle, the vexed question has, according to Carnap, a straightforward answer:

> Frege's system enables us to apply the arithmetical concepts in the description of facts; it enables us to transform a sentence like "the

> number of fingers on my right hand is 5" into a form which does not contain any arithmetical terms. (Carnap 1962: 17)

HP supplies the link between the arithmetical and the empirical because it introduces the notion of number as applicable to (empirical) concepts in the first place. Arithmetic built on HP therefore comes with transformation principles which allow arithmetical theorems to apply to everyday uses of numbers; as Carnap says, it is via HP that 'arithmetical equations, such as $3 + 6 = 9$, become directly applicable to real life situations' ([1939] 1970: 45).[12]

It is worth stressing that the Carnapian does not maintain that HP tracks something psychological about the ways in which humans have always used numerals, or about the meanings of arithmetical concepts in natural language. Such claims, if they are to be rationally debatable, require significant further clarification, namely of the kind a linguistic framework for psychology or empirical linguistics might provide. Rather, the Carnapian does, in the current context, two things only. First, they argue that arithmetic, when it is incorporated into a scientific framework through HP, does not raise philosophical issues concerning applicability.[13] Second, they recommend scientific frameworks which incorporate arithmetic through HP exactly because they avoid applicability questions. The recommendation is made, characteristically enough, on pragmatic grounds: because the recommended frameworks do not give rise to applicability questions, one of those 'wearisome controversies' in the philosophy of mathematics is sidestepped.

What about the value of logicism, by which I now mean: rendering arithmetic *analytic*? Here the Carnapian perspective will differ most drastically from that of the traditional neo-Fregean. For the Carnapian, analyticity is an obscure notion that will only bear serious philosophical

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[12] Frege also believed applicability to be important (e.g. in *Grundgesetze* ([1893/1903] (2013): §159)); in fact, according to Carnap, Frege 'had found his explication of cardinal numbers by asking himself the question: What does "five" mean in contexts like "I have five fingers on my right hand"?' (1963: 47). The Neo-Fregeans also emphasise the importance of a definition of number which has applicability built in, so to speak. Wright (2000) calls this *Frege's Constraint*. See also Cook (2009) and the discussions of Frege's Constraint in Ebert and Rossberg (2017).

[13] In 'Mr Strawson on logical theory' (1953: 446), Quine also mentions Frege's definition of number as an example of elimination of 'philosophical perplexities', in particular of the kind of elimination where a philosophical problem 'turns out not to arise in science as reconstituted with the help of formal logic'. Although the text does not make it clear which philosophical perplexity Frege's definition of number is supposed to have eliminated, it is not implausible that Quine had the applicability problem in mind.

fruit when it is given precision. And it can be given a precise meaning—a precise notion of evidence for statements of the form '*x* is analytic'—relative to a linguistic framework. There are many interesting interpretational questions about Carnap's notion of analyticity, but for the moment we may simply say that a sentence is analytic relative to a linguistic framework if and only if that sentence is an axiom or follows by the framework's logic from the axioms. Of course there is no question about the correctness of this definition, it is simply the definition the Carnapian wants to work with. If all is well, the definition can be shown to do some interesting work. (The more traditional neo-Fregean, although perhaps agreeing that analyticity in its ordinary use is objectionably imprecise, does appear to believe that there is a way of making this notion precise such that HP is, *independently of any framework*, either analytic of the notion of *number* or not (Hale and Wright 2001: 18).)

The question was what the value of rendering arithmetic analytic would be from the Carnapian perspective. The question, we now see, must be reformulated: what is the value of a linguistic framework in which arithmetic is rendered analytic by taking HP as an axiom?

The answer to this question is more complicated, but we can again draw inspiration from Carnap, who emphasises the epistemological clarity of logicism (1963: 47). Unlike the Hilbertian formalist and the Brouwerian intuitionist, the Fregean-Russellian logicist does not acknowledge a special Kantian faculty of arithmetical intuition (Kitcher 1979; Russell [1903] 2010: §4). Such arithmetical intuition sits awkwardly with the empiricism associated with the Vienna Circle, hence Carnap's own attraction to logicism. But what does the epistemological clarity of logicism consist in exactly?

Whereas Frege hoped to secure for arithmetic the same certainty that he believed logic to enjoy, the Carnapian's rejection of arithmetical intuition goes into the other direction: arithmetic is no more certain than our knowledge of the physical world. The Carnapian embraces holism and thus rejects the distinction between the analytic which is beyond doubt and the synthetic which is not. In fact, the Carnapian's analytic-synthetic distinction is not an epistemological distinction at all (Bentley and Uebel 2024). It is rather a distinction within formal semantics, to be drawn only in the presence of a linguistic framework. Of course it may still have epistemological import once we reach what Carnap called *pragmatics*, which includes the study of our justificatory practices and knowledge communication, as the following example will help illustrate.

Imagine a linguistic framework which contains both arithmetic and some general principles of a part of physics, and a scientist working with a more specific theory within this framework—for the moment we leave it open which of the relevant sentences are to count as analytic and synthetic. The scientist's theory predicts that a certain observation sentence is true, but experimentation shows that it is false. With holism, we know that the scientist is in principle free to revise any of the sentences that they accept (as long, of course, as the revised collection of sentences can accommodate the falsified prediction). They might reject the relatively local hypothesis which prompted their prediction, or a more general law of the relevant domain of physics, or arithmetic, or even the law of non-contradiction. The choice is logically arbitrary and potentially overwhelming.

Crucially, it is the analytic-synthetic distinction which will reflect the nature of the possible choices. The sentences which are taken to be analytic come, so to speak, with the linguistic framework: to give up on them is to give up on the framework. They cannot be justified—and therefore also not discredited—theoretically, that is, within the framework. Adopting or giving up on a framework is a matter of practical decision, not a matter of truth. The sentences which are taken to be synthetic are not settled by the axioms and logic of the framework and so must be settled or justified in some other way. In the imagined example, they will be justified—directly in the case of basic observation sentences and indirectly in the case of the more general physical principles—through observation. If the scientist works within a framework in which arithmetic is analytic, then their falsified prediction cannot discredit arithmetic, or show that arithmetic must after all be false. Of course the scientist is free to develop and work with a new framework in which arithmetic is not analytic, or even false, but the proof would be in the pudding: the new framework is to be judged on its performance and, just as the old one, cannot be criticised for getting arithmetic wrong. Such a shift of framework would in some sense have been informed theoretically: it is the falsification of the internal, theoretical hypothesis which we are imagining to have prompted the shift. But the framework shift does not aim at truth. It aspires to utility only.

The example shows that the framework-relative notion of analyticity separates the sentences of a framework that can be falsified theoretically from those which cannot: the former are synthetic, the latter are analytic (Hylton 2019). And in the construction of a linguistic framework, which sentences we take to be analytic is a matter of expediency: there is no absolutely correct or incorrect way of drawing the framework-relative analytic-synthetic distinction.

Where should we now look for the pragmatic value of having arithmetic be analytic within a scientific framework? Thomas Ricketts (2003: 272) has suggested that it is expedient to include arithmetic among the analytic sentences because it reduces 'the scope for the logically arbitrary choices in theory construction that [holism] forces on empirical science' (Ricketts 2003: 272). In the example above, the scientist faces a difficult choice: which of the sentences should they revise in the light of the failed prediction? As noted, only the synthetic sentences of a framework can be falsified by failed predictions; in fact, with holism, they are all discredited together by any failed prediction. The analytic sentences, which partly constitute the framework, can only be rejected on pragmatic grounds: they are not on the table for *theoretical* revision. So if arithmetic is analytic, we make the scientist's choice a little easier by withdrawing arithmetic from their revisionist considerations—so long, at least, as they decide to continue to work with their current framework.

For Ricketts, it is the fact that arithmetic, when it is rendered analytic, cannot be *discredited* theoretically, which pragmatically justifies rendering it analytic. But the epistemological merit that Carnap envisions for logicism lies, I think, rather in the fact that arithmetic, when it is rendered analytic, cannot be *justified* theoretically. As Carnap puts it,

> If we regard interpreted mathematics as an instrument of deduction within the field of empirical knowledge rather than as a system of information, then many of the controversial problems are recognized as being questions not of truth but of technical expedience. The question is: Which form of the mathematical system is technically most suitable for the purpose mentioned? Which one provides the greatest safety? ([1939] 1970: 50)

If arithmetic is included among the synthetic sentences—reckoned a 'system of information'—then all of it, even its axioms (for example HP), must somehow be justified theoretically within the framework. This leads to the awkward question—awkward because so exceedingly elusive as to become irresolvable—how the axioms can be justified internally; arithmetical intuition begins to creep into the picture. If arithmetic is included instead among the analytic truths—reckoned 'an instrument of deduction'—then there is no question of internal justification and only questions of utility remain. The crucial point is that this perspective on arithmetic within a scientific framework—taking it as analytic and hence justified pragmatically—is the most useful one to have because, like building arithmetic on HP in the first place, it obviates the need for some of the often fruitless discussions in the epistemology of mathematics.[14] Such is the value—the epistemological clarity—of logicism.

These reflections, when applied to what is sometimes called the *epistemic* version of the bad company objection, reveal how stark the contrast between the Carnapian and the traditional neo-Fregean is. In setting up this epistemic version, Philip Ebert and Stewart Shapiro (2009) assume that the neo-Fregean, in their answer to the bad company objection, will put forward some conditions which determine the acceptable abstraction principles. Whatever these conditions are (modesty, strong stability, etc., see §1), we may always ask, given some abstraction principle, what we 'have to know or at least be justified in believing about [these conditions] before [we] can be credited with knowledge of the [proposed principle]?' (2009: 424). Must we be able to *prove* that the proposed principle, for instance HP, meets the conditions? Or does it only matter that the principle does in fact meet the conditions, regardless of whether we can prove that it does? Perhaps there is some intermediate position which gets it right. The question, in short, is this: when is our belief in HP, or in any given abstraction principle, warranted?

To the traditional foundationalist neo-Fregean, these are difficult questions. If HP is the epistemological foundation on which our arithmetical knowledge rests, then questions concerning the epistemic status of the foundation are to be taken seriously. The Carnapian, on the other hand, loses no sleep. Although HP is, within their preferred frameworks, the 'foundation' of arithmetic, the principle itself is taken to be analytic and so does not stand or fall with any theoretical justification. Its incorporation among the truths of the framework is justified pragmatically. Of course it might still be useful to prove that HP (or any other given abstraction principle) is modest, or strongly stable, or meets any of the other criteria mentioned in §1, but not because such proofs would theoretically warrant our use of HP. Avoiding traditional epistemology of mathematics is what motivates the Carnapian's logicism (Friedman 2006).

[14] Benjamin Marschall (2024: §2) also stresses that Carnap will 'dismiss many foundational debates in the philosophy of mathematics'.

5 The Carnapian Way Out of Good Company and Duds

For the Carnapian neo-Fregean, the bad company that HP keeps is easily dealt with on pragmatic grounds. In short, it is expedient to have HP be analytic within a scientific framework, and this appears not to hold for its bad company. If this company does turn out to have some interesting use, then of course it is worth investigating; pending such use, the Carnapian simply embraces arithmetic as built on HP without worrying over the bad company.[15]

The Carnapian can deal with the *good* company objection, recently developed by Paolo Mancosu (2015; Sereni, Sforza Fogliani and Zanetti 2023), in much the same way. The good company that HP keeps are abstraction principles from which one can also derive Peano Arithmetic yet that diverge from HP on the transfinite cardinalities. There is, for example, an abstraction principle which entails that the number of Fs is *not* the number of Gs when there are infinitely many Fs, infinitely many Gs, infinitely many non-Fs and finitely many non-Gs (Mancosu 2015: 385–86). HP of course entails (assuming a countable domain) that, in such a case, the number of Fs is the number of Gs. The good company objection challenges the neo-Fregean to say what distinguishes HP from these other abstraction principles that, like HP, do not on their own lead to contradiction yet also yield Peano Arithmetic. Why should arithmetic be built on HP and not on one of these other principles?

The Carnapian will once again emphasise pragmatic value. In the finite cases, the utility of HP coincides with that of its good-company siblings; in particular, the applicability of arithmetic is explained in the same way. If one of these siblings has a use that HP lacks because of their divergence on the infinite cardinalities, then we may embrace it, either within a framework that also contains HP (so long as they are taken to introduce different term-forming operators, or different notions of

[15] Of course the Carnapian also needs to address the Julius Caesar objection mentioned in the introduction—although I cannot do this topic justice here, it might be worth indicating a resolution. In 'Überwindung der Metaphysik durch logische Analyse der Sprache' (1931), before he adopts the principle of tolerance, Carnap simply argues that sentences such as 'Julius Caesar is a prime number' are meaningless. A tolerant Carnapian might instead argue that the typical rules for arithmetical and physical frameworks do not tell us how to deal with such mixed sentences, and so it is up to us to decide, on pragmatic grounds, how to handle them, should we feel the urgency to do so.

number), or as part of a new framework.[16] If there is no use for the sibling principles, there simply is no reason to worry over them. This is of course not a defence of HP over the other candidate principles. Each of these principles allows the derivation of PA and each of them clarifies the connection between arithmetic and the empirical world which the Carnapian stresses. So if there is no reason to care about the transfinite cases, which is where they disagree, then it does not matter which we work with, and we might even opt for a finite version of HP which makes no pronouncements as to the cardinalities of infinite concepts (Heck 1997a).[17]

This characteristically relaxed attitude will once again differentiate the Carnapian neo-Fregean from the more traditional Hale and Wright, who write that

> whatever other virtues are necessary in an acceptable abstraction, [the stipulation of an abstraction principle] cannot be well motivated unless it is unmatched—unless, that is, there is no other abstraction incompatible with it which has exactly the same other virtues. (2001: 426)[18]

Two abstraction principles, equally virtuous yet incompatible, which also serve the same function—the situation could hardly trouble a Carnapian less. The Carnapian endeavour—to construct precise frameworks that facilitate scientific inquiry and rational debate—requires of us simply that we choose one or the other principle but not both. Since we have assumed there to be no interesting difference between them, it does not matter which choice we make, although it might be nice to coordinate our choices if we are to work together.

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[16] The first solution is not so straightforwardly available when two abstraction principles are both useful yet not mutually satisfiable, as for instance with HP and the nuisance or parity principles (assuming for the moment that these latter principles are useful in some context or other). In such cases, the Carnapian might say that the two different goals which these conflicting principles serve require separate frameworks—it might then be an interesting task to find compatible abstraction principles which could serve the same goals at least equally well. Ultimately, this would have to be assessed case-by-case; given the pragmatic and relativistic elements of the Carnapian's position, this should not come as a surprise.

[17] The good company objection stings a lot more if the question is: which of HP and its good company underlies our ordinary notion of cardinality? This question is of course not on the Carnapian's mind (see the second paragraph of §4 above).

[18] Cook (2012: 694) offers his technical notion of SLSC-conservativeness as a formal counterpart to this informal idea of unmatchedness.

A yet different kind of problem that also revolves around the company HP keeps is rather ontological in nature (Hawley 2007: 239–40; Heck 2017; 2000; Hale and Wright 2001: 423). There are infinitely many abstraction principles, both within and without mathematics, that do not lead to contradiction (not even jointly). Hale and Wright (2001: 324), following Heck (2000), mention the abstraction principle which introduces the 'dud' of an entity via the equivalence relation

> x = one of Heck's shoes or x = Queen Victoria or x = the blackboard in 104 Emerson Hall and y = one of Heck's shoes or y = Queen Victoria or y = the blackboard in 104 Emerson Hall.

Similarly contrived equivalence relations are easily concocted. Unless the neo-Fregean is happy to accept all such artificial principles and so inflate their ontology massively, for instance with weird entities called 'duds', they have to explain what distinguishes HP, which they argue should be accepted, from the abstraction principles that introduce unwanted entities and so presumably should not be accepted.

A crucial element of the Carnapian perspective is the almost total disregard for ontological worries. 'Aren't you concerned that you must admit, via all the consistent principles of the same form as HP, the existence of abstract entities that you cannot possibly want in your ontology?'. The Carnapian cannot help but be surprised by—or, in the current metaphysical climate, weary of—such ontological anxiety. It is not true, after all, that the abstract objects weigh us down as we try our best to make a living and a life. The philosopher who is committed to many abstract objects does not get back pains from carrying their abstracta with them, and the philosopher who accepts next to no abstract objects does not because of that fact get to tread especially lightly on the pavement. Why, then, worry about them? Because there is no external perspective from which a framework can be judged to be correct or incorrect, there just is no reason to be afraid of abstract objects. If they are useful, use them; if not, not.

6 A Comparative Conclusion

The Carnapian perspective on neo-Fregeanism I have sketched is not altogether new.[19] Alan Weir, for instance, briefly considers a relativist version of neo-Fregeanism according to which abstraction principles are to be evaluated only relative to a particular proof system (2005: 344–46). How are the proof systems to be evaluated? On the one hand, Weir's relativist seems to think they should be assessed on their coherence and utility, which is music to the Carnapian's ears. On the other hand, Weir's relativist seems to be relativist only because they believe that there is no unique proof system which 'corresponds to the real, mind-independent, universe'. The Carnapian of course rejects such a metaphysical notion altogether, at least if it is intended to be something external and absolute. Carnapian relativism is not motivated by the denial of some metaphysical claim to the effect that there is a unique structure to the universe; rather, Carnapian relativism is the rejection of that kind of metaphysics.

Another example of nascent Carnapian neo-Fregeanism comes from Agustín Rayo's work. Rayo considers what he calls a 'trivialist' version of neo-Fregeanism according to which Hume's Principle is what he calls a 'just is'-statement: for the number of Fs to be the number of Gs *just is* for there to be a bijection between (the extensions of) F and G (2013: 77; see also Rayo 2017). Moreover, he has admitted to being Carnapian in the sense that he does not believe in a 'notion of *objective* truth or falsity' for 'just is'-statements (2014: 525–33). Instead, he thinks that such statements should be accepted or rejected 'depending on whether they lead to fruitful theorizing' and that '*relative* to the set of "just is"-statements one accepts the statements in that set will count as true' (2014: 527). Insofar as Rayo's 'just is'-statements are similar to the analytic sentences in a Carnapian framework, he really is very Carnapian. He does not, however, embrace a Carnapian neo-Fregeanism—only calling trivialist neo-Fregeanism 'the most interesting version of [neo-Fregeanism]' (2013: 78)—and neither does he consider any reasons for thinking that HP, as opposed to the Peano axioms, leads to especially 'fruitful theorizing', which is where the Carnapian has, as we have seen, so much to say.

Yet another example of a Carnapian-looking position is that of Aldo Antonelli (2010). According to his deflationary stance, the point of

[19] See, besides the three mentioned, also Amie Thomasson's easy approach to ontology (2015: ch. 3). My Carnapian, unlike hers, strongly emphasises the relative and pragmatic nature of the Carnapian approach.

abstraction principles is simply to impose ‘a lower bound on the cardinality of [some domain], relative to the size of the space of concepts over [that domain]’ (2010: 201). In particular, the introduction of such a lower bound through an abstraction principle need not be accompanied with the installation of a ‘separate, privileged ontological ream’ of abstract objects. Since no metaphysics is implied, metaphysics and metaphysics-adjacent epistemology—how do we have access to these metaphysically spooky entities we call ‘numbers’?—do not decide which abstraction principles are good and which are bad. So how should we decide? Antonelli’s answer is pragmatic in nature: the good principles are simply the ones that ‘have turned out to be useful in some context or other’ (2010: 201). The similarities with the Carnapian are obvious.

So what is unique about the Carnapian perspective? The dogged insistence that many vexed issues in the epistemology and metaphysics of mathematics are better avoided than addressed—this is what sets the Carnapian apart (cf. Friedman 2006). Although Rayo and Antonelli appear to agree that abstraction principles are to be judged primarily on their relative expediency, it is only the Carnapian who persistently emphasises the dispelling of traditional epistemological conundrums as the most fruitful consequence of the analyticity of HP.

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