A Percolation Model for Infidelity in Monogamous Systems with Contagious Kinks

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Abstract

This paper introduces a mathematical model to analyze the stability of monogamous systems under the influence of contagious proclivities (referred to as "kinks") using a jigsaw percolation framework on graphs. We model a population as a graph where vertices represent individuals, and edges denote relationships or proclivity compatibility. Monogamy is represented as a perfect matching, with infidelity modeled as additional edges. We incorporate the concept of contagious kinks, where proclivities spread through compatible connections, and explore conditions under which infidelity does not occur, i.e., the system remains stable with clusters of size at most two. Using extremal graph theory (Turán's theorem) and network flow constraints, we derive thresholds for the probability of extra-relational edges to prevent percolation. We further account for population growth via reproduction and the amortized cost of contagion, showing that kink suppression (reducing compatibility probability) significantly delays the critical population size at which infidelity becomes inevitable. Our results suggest that monogamy can be preserved if the probability of extra-relational edges remains below a critical threshold, even with contagious kinks, provided the graph remains fragile and network flow constraints limit relationship concurrency.

Introduction

Monogamy, as a social structure, assumes exclusive pairwise relationships, but real-world systems often face challenges from infidelity, which can destabilize the structure. We propose a novel approach to study this phenomenon using jigsaw percolation [1], a model originally developed to study cooperative problem-solving in social networks. Here, we adapt it to model the spread of contagious proclivities ("kinks") in a monogamous population, where infidelity is represented as the formation of larger connected components (clusters) in a graph.

We define two graphs: G_{people}, representing social or romantic relationships, and G_{puzzle}, representing compatibility or transmissibility of kinks. Kinks are assumed to be contagious, meaning they spread through compatible connections. The system remains monogamous if clusters in the jigsaw percolation process remain of size at most two (i.e., limited to monogamous pairs). We use Turán's theorem [2] to constrain the formation of larger cliques (e.g., triangles, representing group interactions) and network flow theory to enforce capacity constraints, preventing concurrent relationships ("multithreading"). We also consider population growth via reproduction and the amortized cost of kink contagion, exploring whether a critical population size exists where monogamy collapses, necessitating a halt in reproduction.

Model Description

We model a population of n individuals (assumed even for simplicity) as vertices in two graphs:

- $G_{\text{people}} = (V, E_{\text{people}})$: Vertices represent individuals, and edges represent relationships. In a pure monogamous system, G_{people} is a perfect matching M with $|M| = n/2$ edges, where each vertex has degree 1. Extra-relational edges (infidelity) are added with probability p_{extra}, forming E_{extra}.
- $G_{\text{puzzle}} = (V, E_{\text{puzzle}})$: Edges represent kink compatibility. Within monogamous pairs in M, edges exist with probability $p_{\text{kink}} \approx 1$ (fixed kinks). Between non-paired vertices, edges exist with probability $p'_{\text{kink}} = c \log n / n$, reflecting lower compatibility.

Jigsaw Percolation

In jigsaw percolation [1], clusters merge if there exists at least one edge in both G_{people} and G_{puzzle} between them. Starting with singleton clusters, the process continues until no further merges are possible. Monogamy is preserved if all clusters have size at most 2. Percolation occurs if a giant component (size $\Theta(n)$) forms, indicating widespread infidelity.

Fragile Graph

We define G_{people} as fragile if the number of extra-relational edges $|E_{\text{extra}}| < n/2$, keeping the graph below the connectivity threshold of random graphs [3]. This ensures limited connectivity, preventing large clusters.

Network Flow and Non-Multithreading

Each vertex has a capacity of 1, reflecting one stable relationship. Edges in M have capacity 1 (saturated), and extra-relational edges cannot carry flow due to saturated vertex capacities, preventing concurrent relationships.

Population Growth and Amortized Cost

Population grows as $n(t) = n_0(1 + r)^t$, where r is the reproduction rate (e.g., $r = 0.1$). The amortized cost of kink contagion is the number of active edges in G_{puzzle} per vertex:

$$\text{Cost} = \frac{|E_{\text{puzzle, active}}|}{n}$$

As n grows, $|E_{\text{puzzle}}| \approx \binom{n}{2} p'_{\text{kink}}$, increasing the cost.

Kink Suppression

Suppressing kinks reduces p'_{kink} to $\alpha p'_{\text{kink}}$ (e.g., $\alpha = 0.5$), modeling a cultural or perceptual shift reducing kink transmissibility.

Mathematical Analysis

We derive conditions for no infidelity (no percolation, clusters of size ≤ 2).

Theorem 1. Infidelity does not occur if:

$$p_{\text{extra}} \times p'_{\text{kink}} \leq \frac{1}{c\,n\,\log n}$$

and $|E_{\text{extra}}| < n/2$, with vertex capacities limited to 1.

Proof. For no percolation, the probability of merging two clusters (e.g., pairs $\{u, v\}$ and $\{w, x\}$) is:

$$P(\text{merge}) = p_{\text{extra}} \times p'_{\text{kink}}$$

The expected number of merges is $\binom{n/2}{2} \times p_{\text{extra}} \times p'_{\text{kink}}$. For no giant component [1]:

$$p_{\text{extra}} \times p'_{\text{kink}} \leq \frac{1}{c\,n\,\log n}$$

With $p'_{\text{kink}} = c \log n / n$, we get:

$$p_{\text{extra}} \leq \frac{1}{c^2 n (\log n)^2}$$

For fragility, $|E_{\text{extra}}| \approx \binom{n}{2} p_{\text{extra}} < n/2$. In network flow, vertex capacity 1 ensures no augmenting paths for extra-relational flow, preventing infidelity.

Turán's Theorem

To prevent K_3 (threesomes), G_{people} must have at most:

$$ex(n, K_3) = \frac{n^2}{4}$$

edges [2]. With $|M| = n/2$, extra edges must satisfy $|E_{\text{extra}}| \leq n^2/4 - n/2$.

Population Growth

With $n(t) = n_0(1 + r)^t$, the threshold becomes stricter as n grows. For p_{extra} fixed, there exists a critical n_c where percolation becomes inevitable.

Lemma 1. The critical population size n_c satisfies:

$$p_{\text{extra}} \times \frac{\log n_c}{n_c} \approx \frac{1}{c\, n_c \, \log n_c}$$

Proof. Set $p'_{\text{kink}} = \log n_c / n_c$ and solve:

$$p_{\text{extra}} \leq \frac{1}{c\, n_c \, (\log n_c)^2}$$

For fixed p_{extra}, n_c is the largest n satisfying the inequality.

Kink Suppression

If $p'_{\text{kink}} = \alpha \log n / n$, the threshold becomes:

$$p_{\text{extra}} \leq \frac{1}{c\, \alpha\, n \, (\log n)^2}$$

For $\alpha = 0.5$, n_c increases significantly.

Numerical Example

Consider $n_0 = 100$, $p_{\text{extra}} = 0.01$, $r = 0.1$, $c = 1$.

Without Kink Suppression

- $p'_{\text{kink}} = \log 100/100 \approx 0.046$ (base e).
- Threshold: $p_{\text{extra}} \leq 1/(100 \times 4.6 \times 0.046) \approx 0.047$. Since $0.01 < 0.047$, no infidelity.
- Amortized cost: Active edges $\approx 50 + 4900 \times 0.046 \approx 275$, cost ≈ 2.75.
- Critical n_c: For $n = 11,739$ (50 generations), $p'_{\text{kink}} \approx 0.0008$, threshold $\approx 0.000011 < 0.01$, so infidelity occurs.

With Kink Suppression ($\alpha = 0.5$)

- $p'_{\text{kink}} = 0.5 \times 0.046 = 0.023$.
- Threshold: $p_{\text{extra}} \leq 1/(100 \times 4.6 \times 0.023) \approx 0.094 > 0.01$, still no infidelity.
- Cost: Active edges $\approx 50 + 4900 \times 0.023 \approx 163$, cost ≈ 1.63.
- Critical n_c: Solve $0.01 \times (0.5 \log_{10} n_c / n_c) \leq 1/(n_c \log_{10} n_c)$, yielding $n_c \approx 10^{460}$.

Conclusion

Our model shows that monogamy is preserved if p_{extra} is below the percolation threshold, the graph remains fragile, and vertex capacities prevent concurrent relationships. Population growth increases the amortized cost of contagion and tightens the threshold, potentially leading to a critical $n_c \approx 11,739$ without kink suppression. Suppressing kinks (e.g., $\alpha = 0.5$) drastically increases n_c to $\approx 10^{460}$, delaying the need to halt reproduction. Future work could explore dynamic p_{extra} or heterogeneous kink distributions.

References

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